

Generalization of the MTU 3-D Crack Propagation Criterion for anisotropic materials

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Summary: In this article, the MTU crack propagation criterion is generalized and applied to anisotropic materials with arbitrary crack orientations. Qualitatively, there is a good agreement with the isotropic results, however, the symmetry in the solution disappears for arbitrarily oriented cracks. Furthermore, the differences with the Paderborn criterion are slightly larger, which may allow for an experimental verification.

Keywords: crack propagation criterion, mixed-mode, anisotropic materials

Verallgemeinerung des MTU 3-D Rissfortschrittskriterium auf anisotrope Materialien

Zusammenfassung: In diesem Artikel wird das MTU Rissfortschrittskriterium verallgemeinert und auf anisotrope Materialien angewandt. Qualitativ stimmen die Ergebnisse für anisotrope Materialien mit denen für isotrope Materialien überein, für beliebig orientierte Risse fallen jedoch die Symmetrien in der Lösung weg. Die Unterschiede mit dem Paderborner Kriterium sind außerdem etwas größer, was eine experimentelle Verifizierung möglich erscheinen läßt.

Stichwörter: Rissfortschrittskriterium, mixed-mode, anisotrope Materialien

1. Introduction

By now, automatic crack propagation software is readily available[1]. One of the key aspects of such software is the crack propagation criterion. It predicts the crack propagation rate and direction based on crack driving parameters such as the stress intensity factor K , the J -integral or the Crack Opening Displacement. Here, we focus on linear elastic crack propagation calculations using the K -concept. Generally, there are three crack modes: mode I, which is a tension mode, and modes II and III, which are shear modes orthogonal to the crack front and parallel to the crack front, respectively. For each of these modes a K -factor can be determined and it is the task of the crack propagation criterion to calculate the crack propagation rate and direction based on these three values. Several crack propagation criteria have been proposed in the past [2], however, most of these are based on 2-D considerations. Recent 3-D developments include a criterion developed by Dhondt at MTU [3] and one created by Richard and co-workers at the University of Paderborn [4].

In [3] the MTU 3-D crack propagation criterion has been explained and applied to isotropic materials. It is characterized by the straight use of the idea that cracks propagate orthogonal to the maximum principal asymptotic stress. Quintessential is in this respect the requirement that the crack propagation plane must contain the crack tip (crack tip inclusion requirement). The Paderborn criterion is based on the maximal principal stresses of the stress tensor projected on a cylindrical surface sur-

rounding the crack front. Comparison of the MTU criterion with the Paderborn criterion has shown good agreement for the usual test arrangements, however, there are differences for pure Mode II-III combinations and for combined Mode I-II-III states. In this article, the MTU criterion is generalized and applied to anisotropic materials with arbitrary crack orientations. Furthermore, also negative K-values are taken into account, thus covering the complete K-range. For the first time the situation is encountered that, due to the crack tip inclusion requirement, the crack propagation direction is determined by the second largest principal stress value instead of the largest one. At first, the MTU criterion is explained in detail. Then, it is applied to the complete K-range for isotropic materials. Finally, the criterion is generalized to anisotropic materials and compared with results obtained by the Paderborn criterion.

2. Crack propagation criterion

For a given combination of K_I , K_{II} and K_{III} , the MTU crack propagation criterion assumes that crack propagation will take place in a plane orthogonal to a principal stress of the asymptotic stress field (let's call this for brevity a principal asymptotic stress). Furthermore, this plane must contain the crack tip (so called crack tip inclusion requirement). If several planes satisfy this condition, the plane orthogonal to the largest of the corresponding principal asymptotic stresses is decisive. Now, let's analyze this criterion in detail.

The stress field at the crack tip is singular, its largest term in its series expansion is the so-called asymptotic stress field. Analytical expressions for the asymptotic stress field for isotropic materials can be readily found in the literature [2][5]. The asymptotic stress field is the first term on the right hand side of the following expansion:

$$\sigma(r, \varphi, z) \approx \frac{\sigma^*(\varphi, z)}{\sqrt{2\pi r}} + O(1)$$

Here, σ stands for any stress component, r and φ are polar coordinates with origin at the crack tip. Multiplying the asymptotic stress field with $\sqrt{2\pi r}$, one obtains the self-similar stress field σ^* . Notice that the asymptotic stress field and the self-similar stress field have the same principal directions, the principal values differ by the amount $\sqrt{2\pi r}$. The self-similar stress field actually has the same dimensions as the stress intensity factor.

To explain the crack tip inclusion requirement, let us focus on a two-dimensional configuration, Figure 1. The crack tip is located in the origin and the crack extends to the left along the x-axis. The angle φ denotes a potential crack propagation direction. Any point on the line L is characterized by the same self-similar stress tensor, since the latter does not depend on the distance from the crack tip r . Consequently, the asymptotic stress tensors differ by a constant. This implies that the principal asymptotic stress planes for all points along this line all have the same orientation. This is symbolized by the blue parallel lines, representing the principal asymptotic stress plane corresponding to the principal self similar stress σ_1^* . These planes do not contain the crack tip, no matter how close to the tip one selects a point on line L. Choosing such a plane as crack propagation plane would lead to a discontinuity at the

crack tip. This is unacceptable. Therefore, we require that the crack tip must lie in the crack propagation plane. This condition is exactly expressed by the crack tip inclusion criterion. This means that an angle φ is to be identified, which yields a crack propagation direction φ_0 such that $\varphi = \varphi_0$.

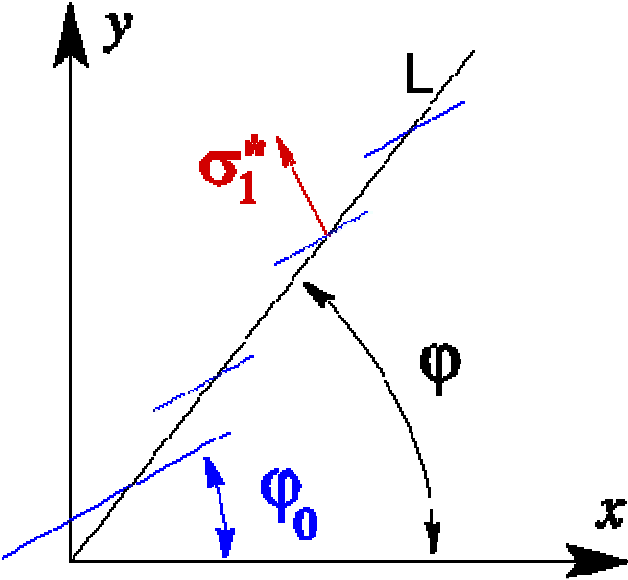


Figure 1: Potential crack propagation direction and principal asymptotic stress planes

The situation may arise, that different combinations of (φ, σ_1^*) satisfy the condition $\varphi = \varphi_0$. This ambiguity is resolved by postulating that the crack propagation plane is determined by the highest value of σ_1^* for which $\varphi = \varphi_0$.

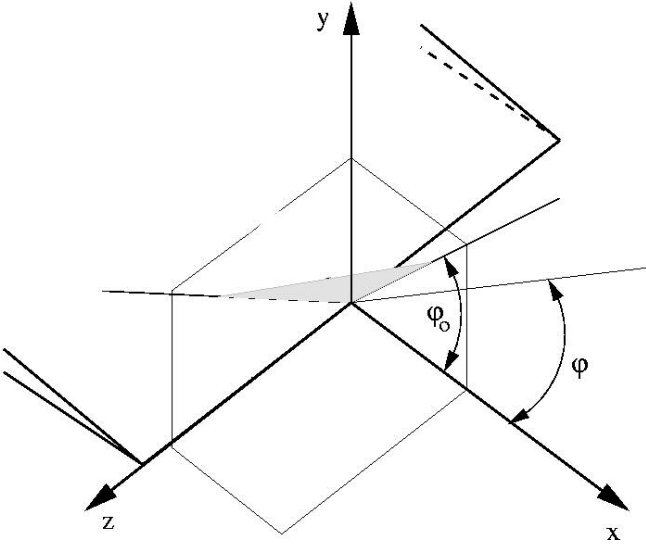


Figure 2: three-dimensional definition of φ_0

For three-dimensional applications a local rectangular coordinate system is defined at the crack tip (Figure 2): the x-axis and z-axis belong to the local crack plane (the x-axis is orthogonal to the crack front, the z-axis is tangential to the crack front) whereas the y-axis is orthogonal to the crack plane. The angles φ and φ_0 are measured in the x-y plane, starting from the x-axis. Now, the angle φ_0 is defined as the angle between the x-axis and the intersection line of the principal asymptotic stress plane and the x-y plane.

To determine the crack propagation direction, the principal asymptotic planes and their corresponding angle φ_0 are evaluated for several values of φ between -90° and $+90^\circ$, Figure 3. As a matter of fact, for each value of φ there are three values of φ_0 , since there are three principal stresses per stress tensor. In Figure 3, only the value corresponding to the highest principal asymptotic stress is shown. The intersection of the φ -curve with the φ_0 -curve yields a potential solution. The solution is found if all other intersection points lead to smaller self-similar stresses.

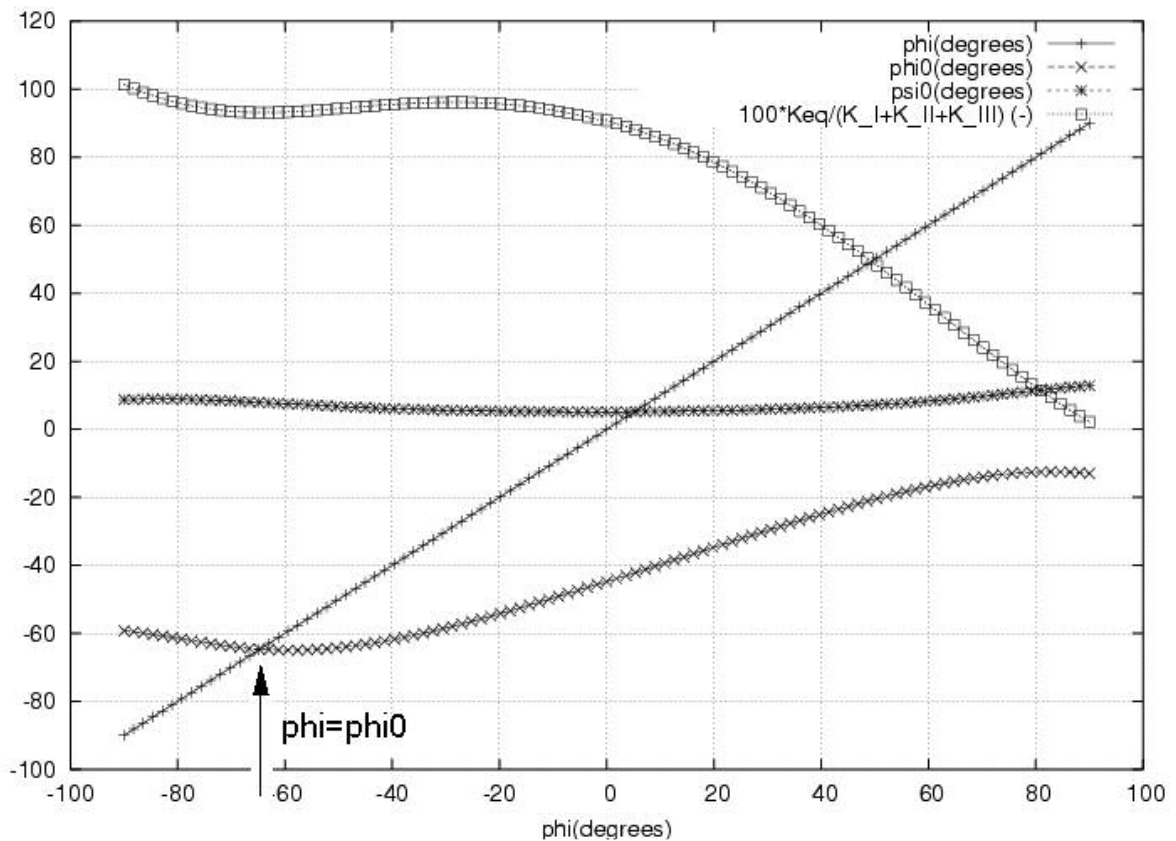


Figure 3: Crack deflection angles and equivalent K-factors as a function of φ

Figure 3 shows the deflection angles and equivalent K-factors for steel with Young's modulus $E=210,000 \text{ N/mm}^2$ and Poisson coefficient $\nu=0.3$ for a K-triplet satisfying $K_I=0.2$, $K_{II}=0.7$ and $K_{III}=0.1$ (the sum of the K-factors is normalized to 1). The first deflection angle is φ_0 , which has already been introduced. The second angle is ψ_0 , which is the angle between the principal asymptotic stress plane and the z-axis (this

angle is measured in a plane orthogonal to the principal asymptotic stress plane and containing the z-axis). The equivalent K-factor is actually the principal self-similar stress σ^* . Indeed, it was already noticed that σ^* has the unit of a stress intensity factor. In a lot of cases it suffices to look at the largest principal asymptotic stress to find the solution. From Figure 3 we learn that the crack propagation angle in this concrete case is about -64° and the normalized equivalent K is 0.95. Notice that the equivalent-K curve does not necessarily have a maximum for $\varphi = \varphi_0$.

3. Extension to negative K-values.

In [3] the representation of the results in barycentric coordinates was explained. It refers to a visualization of all (K_I, K_{II}, K_{III}) combinations with sum 1 in the form of a triangle. All K-values of points within this triangle are positive. This visualization scheme can be easily extended to negative K-values by reflection about the sides of the triangle. This is illustrated in Figure 4.

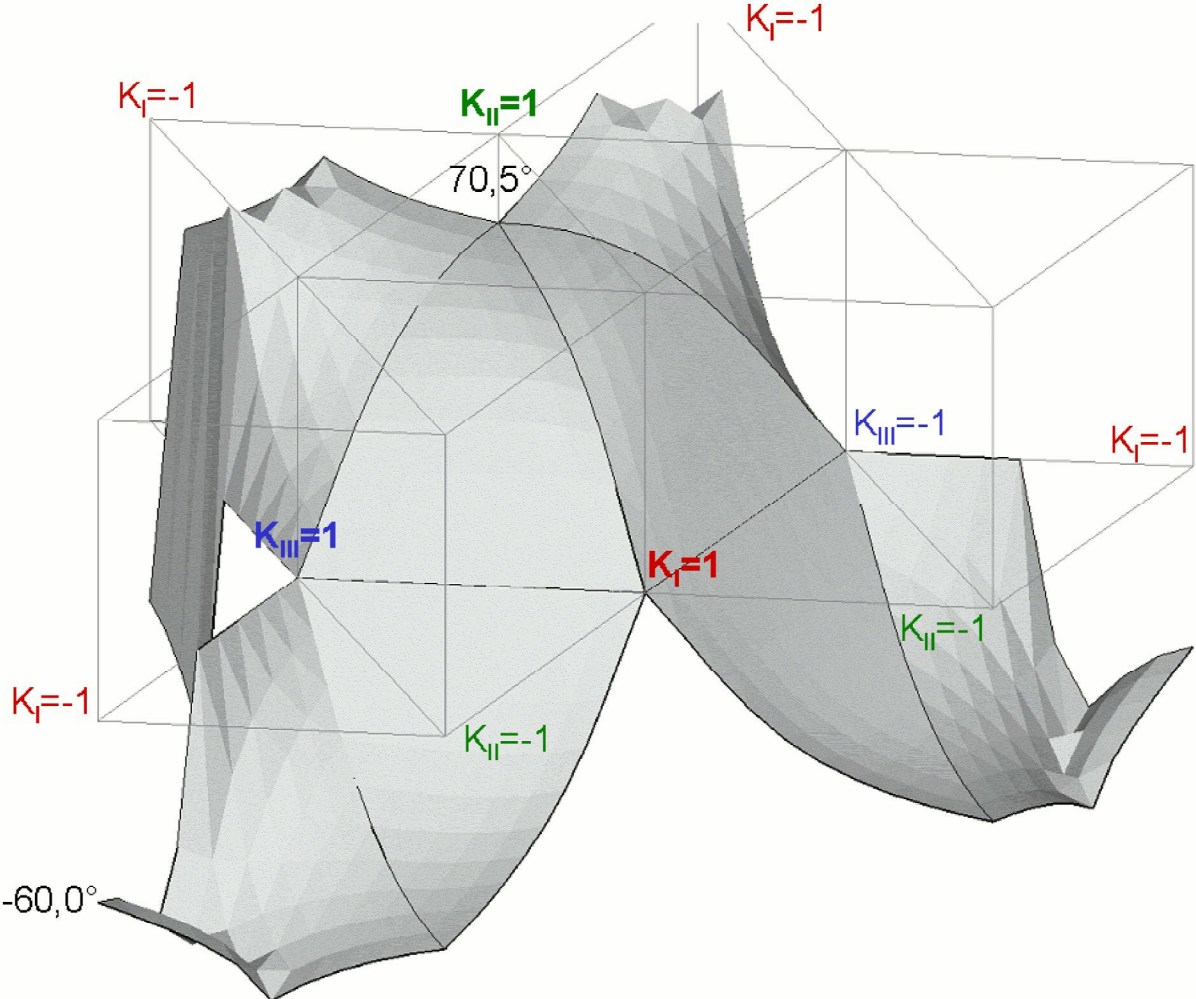


Figure 4: Total representation of $-\varphi_0$ for an isotropic material (similar to the pictures published in [4], $-\varphi_0$ is shown rather than φ_0 ; indeed, φ_0 is negative for positive K_{II} , cf. Figures 1 and 2)

The triangle with corner points $K_I=1$, $K_{II}=1$, $K_{III}=1$ is the basis triangle. By reflection of the point $K_{II}=1$ about the line through $K_I=1$ and $K_{III}=1$, one obtains a triangle for which the K_I and K_{III} -values are positive, whereas the K_{II} -values are negative (the sum of the absolute values of the K 's is still 1). If one proceeds similarly for the other sides one obtains eight triangles covering every possible combination of the stress intensity factors. Negative K_{II} and K_{III} -values do not yield any new information for isotropic materials, however, negative K_I -values do. This is very interesting for applications, since small negative K_I -values are not so unusual. One can argue that for negative K_I -values the crack is subject to pressure and will be closed, necessitating contact considerations. This leads to nonlinear calculations and may jeopardize the use of the K -concept. However, crack surfaces are usually reasonably rough and maybe the contact takes place some small distance away from the crack tip and not immediately at the crack tip. This would allow the use of the K -concept on a microscale. Notice that discontinuities in φ_0 can occur if 2-D or 3-D hydrostatic stress states are crossed. Figure 5 shows the equivalent K -factor for the same application. It exceeds 1 for pure Mode-II applications and decreases smoothly for negative K_I -values. Notice that all K -combinations could also be represented on a globe, with $K_I=1$ and $K_I=-1$ being the opposite poles.

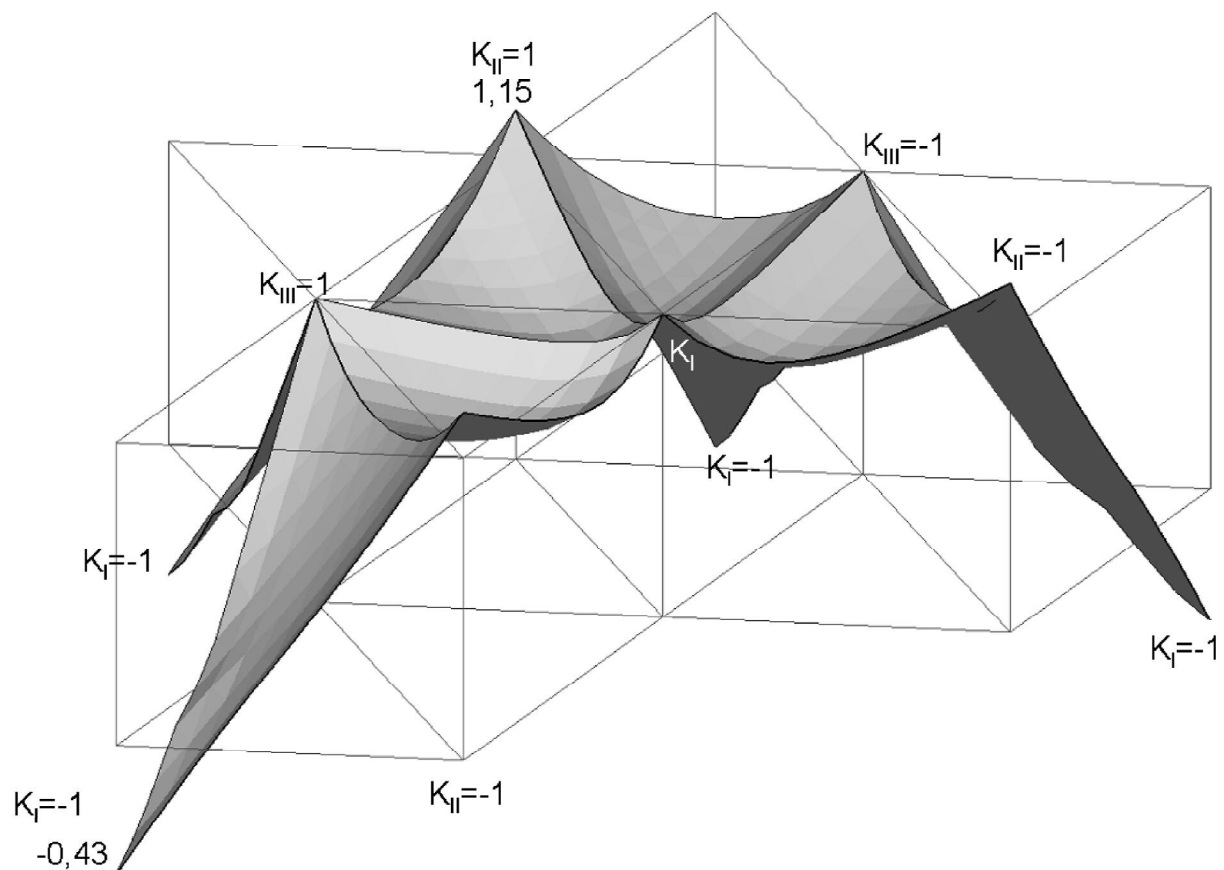


Figure 5: Total representation of the equivalent K -factor

4. Extension to anisotropic materials

For an anisotropic material the procedure is basically the same, however, some of the calculations are more tedious. Indeed, there is no analytical formula for the asymptotic stress field, it has to be calculated numerically [6]. Furthermore, for the first time the phenomenon occurred, that the highest principal asymptotic stress did not yield the crack propagation plane. Recourse had to be taken to the second highest principal stress. This is illustrate in Figures 6 and 7. They show the deflection angles and equivalent K-factor for an orthotropic material with stiffness constants $D_{1111}=D_{2222}=D_{3333}=228,000$ MPa, $D_{1122}=D_{1133}=D_{2233}=140,000$ MPa and $D_{1212}=D_{2323}=D_{1313}=127,000$ MPa. The stress intensity factors took the values $K_I=0.95$, $K_{II}=0.025$ and $K_{III}=0.025$. Figure 6 shows the largest principal self-similar stresses and the corresponding deflection angles. There is no solution to the equation $\varphi = \varphi_0$. Indeed, the discontinuity near $\varphi=0$ is caused by the switch between $\varphi_0=-90^\circ$ and $\varphi_0=+90^\circ$. This switch is smooth, the discontinuity arises due to the definition of φ_0 . Explained in another way, $\varphi_0=-90^\circ$ and $\varphi_0=+90^\circ$ are really one and the same state and consequently there is no crossing with the φ -curve in Figure 6. A solution would have been very surprising anyway since it would mean that the equivalent K-factor exceeds 1 for stress states near Mode-I.

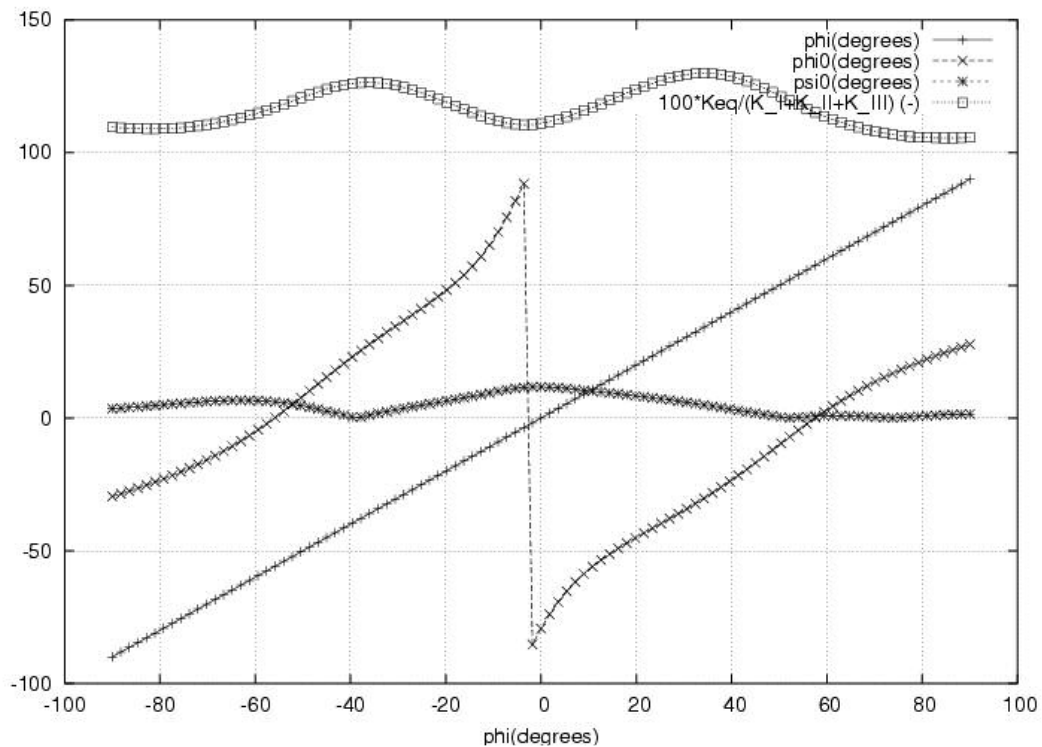


Figure 6: Deflection angles and equivalent K-factor for an orthotropic materials:
Maximum principal asymptotic stress

So we have to look at the middle principal self-similar stress, Figure 7. Here, we do have a crossing near $\varphi=0$ leading to an equivalent K-factor slightly under 1. Notice that here too there are two jumps from $\varphi_0=-90^\circ$ to $\varphi_0=+90^\circ$ or vice versa which do not lead to solutions.

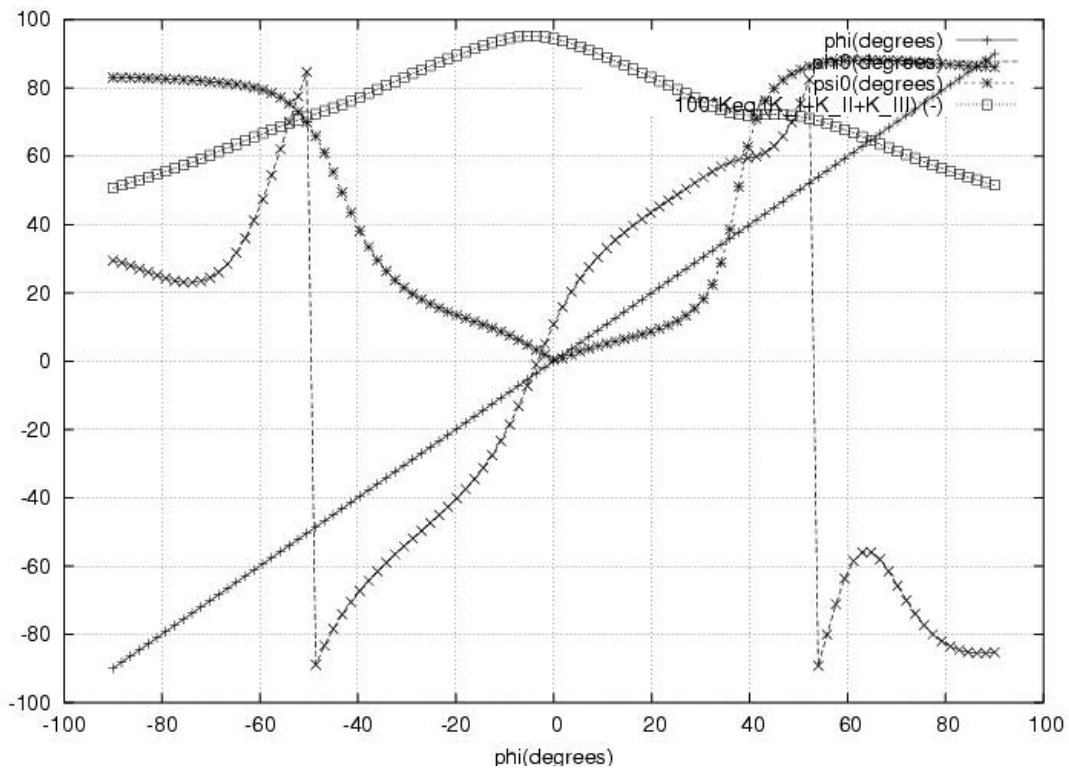


Figure 7: Deflection angles and equivalent K-factor for an orthotropic materials: Middle principal asymptotic stress

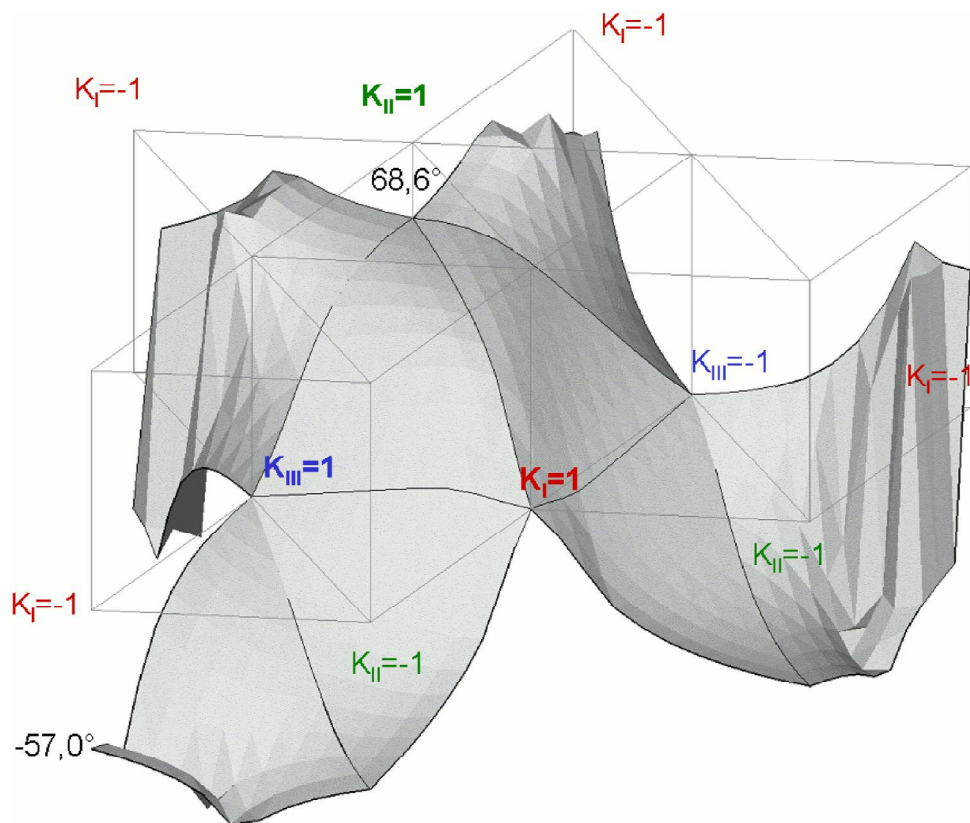


Figure 8: Total representation of $-\varphi_0$ for an anisotropic material

Qualitatively, the results we obtained for anisotropic materials are very similar to the ones for orthotropic materials. Figure 8 shows the first deflection angle. The values are slightly different from those in Figure 4 but the overall behavior hasn't really changed, apart from some loss of symmetry.

In order to compare with the criterion developed at Paderborn [4], the latter was expanded to anisotropic materials too. The Paderborn criterion uses the maximum principal stress corresponding to a projection of the stress state on a cylinder surrounding the crack front. In a previous publication [3] it was shown that the difference for the first deflection angle between both criteria is about 8° . A similar calculation here leads to the values in Figure 9.

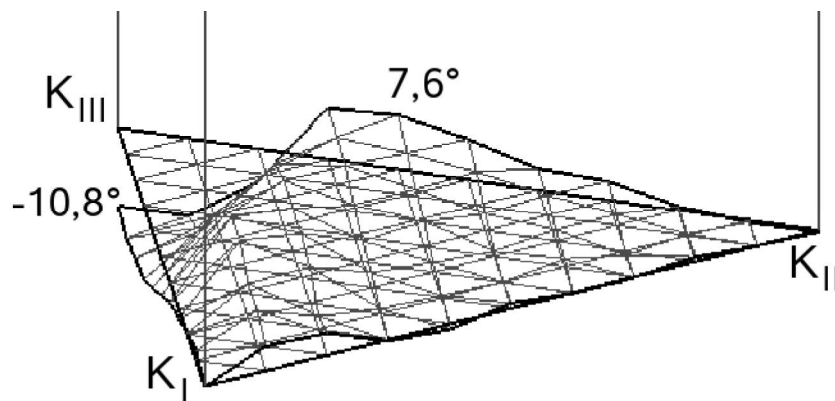


Figure 9: $\varphi_0(\text{MTU}) - \varphi_0(\text{Paderborn})$

The difference is now slightly increased to about 11° (pure Mode-III). Unfortunately, the largest differences occur for mode combinations which are very difficult to verify experimentally. Nevertheless, a project to determine crack deflection angles for mixed-mode conditions is highly needed.

5. Conclusions.

The MTU criterion was extended to negative K-values and to anisotropic materials. A comparison with the Paderborn criterion for anisotropic materials reveals differences in the deflection angles up to 11° for true mixed-mode stress states, especially if mode III is involved. For mixed Mode I – Mode II states the differences are rather small.

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