# Non-Linearities in an Aero-Engine Structure: From Test to Design

**S. Perinpanayagam, D. Robb, D. J. Ewins**
Centre for Vibration Engineering
Mechanical Engineering Department
Imperial College London
Exhibition Road, London SW7 2BX
email: s.nayagam@imperial.ac.uk

**J. Moreno Barragan**
MTU Aero Engines GmbH
Engine Dynamics
Dachauer Strasse 665
D-80995 Munich

## Abstract

There are several features in an aero-engine structure, including the bolted scalloped flange, the rotor attachment to bearings, amongst others, which may give rise to non-linear behaviour at large amplitudes of vibration. Currently in industry, linear finite element models have been used to predict the behaviour of the structure at various force patterns experienced in the operating conditions of the structure. However, these predictions may become invalid if the non-linearities in the structure considerably affect the structure's dynamic behaviour.

In this paper, a testing technique is presented for identifying and characterising the non-linear behaviour of an aero-engine. Furthermore, this technique is used to obtain test data from a real engine structure and the results from the test are discussed with respect to correcting the non-linear elements of the analytical model of the aero-engine structure.

## 1 Introduction

A cutout of a gas turbine engine structure is shown in Figure 1. One of the main characteristics of a large complex practical engineering structure such as a gas turbine engine is that it consists of many components and subassemblies joined together by different types of interfaces. These interfaces between the individual components or subassemblies are important features which contribute considerably to nonlinear behaviour. Such interfaces can be found between the casing using a bolted scalloped flange and between the rotor and the stator using bearings. The critical interfaces of an aero-engine are shown in Figure 2.

Generally, due to the linear finite element models used in industry, these nonlinear features and their behaviour are not taken into consideration when predicting the behaviour of the structure. However, these predictions may become invalid if the non-linearities in the structure considerably affect the structure's dynamic behaviour. In an extreme case, using linear analysis for predicting highly nonlinear modal parameters of a structure could lead to failure of an engine structure in operation. If a valid analytical model for the nonlinear behaviour of the structure needs to be created, an efficient testing and extraction procedure needs to be developed to identify and characterise the nonlinear behaviour of the aero-engine. The modal test, which is based on linear principles, cannot be used directly to obtain information about the nonlinear behaviour of the joint parameters, or to validate the corresponding analytical models. However, the amplitude-dependent spatial parameters of the joints can be characterised using controlled-level vibration (CLV) tests.

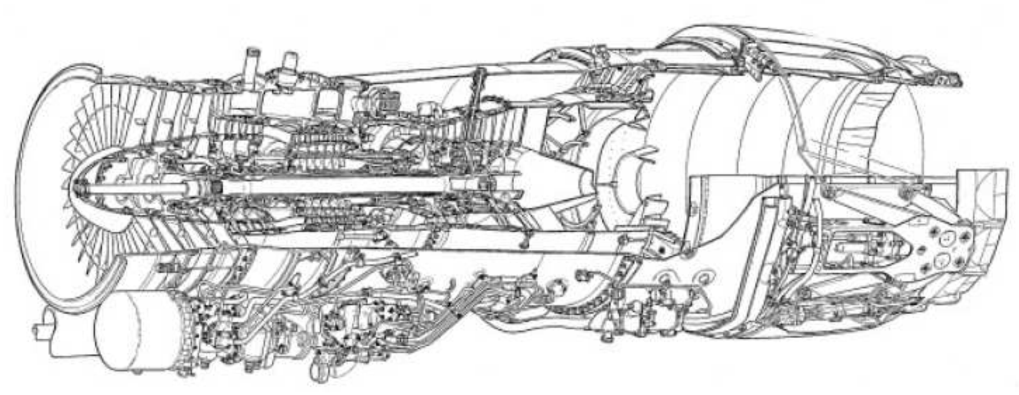


**Figure 1 Cutout of a gas turbine engine**

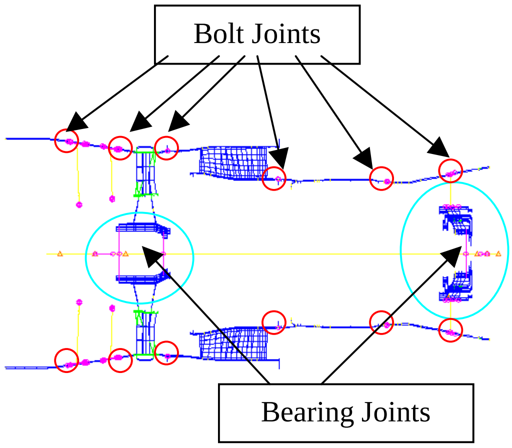



**Figure 2 Different types of joints present in an aero-engine**

In a CLV test, the excitation levels of the sine signal to the shaker(s) are controlled according to certain criteria so that equivalent linear behaviour of the structure can be obtained at this steady-state harmonic excitation level. This is similar to harmonic balance method [Sanliturk and Ewins 1996] employed for obtaining simulated frequency response functions for a structural model with nonlinear elements such as joints. In fact, a CLV test can be described as the matching partner for the harmonic balance method from the experimental standpoint. Sine excitation is chosen in CLV testing due to the controllability, precision, frequency-selective nature and good signal-to-noise ratio with which the structure can be excited.

In the following sections, first the control system for CLV testing is briefly described. Then, such a control system developed in Lab View is used to obtain nonlinear test data from a real aero-engine structure. This test data is further discussed with respect to correcting the non-linear elements of the analytical model of the aero-engine structure.

## 2 Testing Techniques for Nonlinear Characterisation

There is literature available on control system designs for CLV testing [Bucher 1998; Ferreira 1998]. These studies use an iterative off-line control to tune the excitation forces to fulfil certain criteria. The controls are also performed in the frequency domain rather than the time domain. According to Bucher *et al.* [1995], the active control using the time domain lacks the precision provided by off-line (or feed forward) steady state based methods because of the constant signal adaptation. There are three main key considerations that need to be incorporated in the control system design for CLV testing.

1. The control system should keep the force level constant at resonance. This is because when the structure under test resonates with large relative displacements, the exciter interacts with the structure, and the reaction force between the exciter and the structure tends to become very small. This phenomenon is called force drop off in vibration testing and it has been discussed by several authors [Olsen 1986; Rao 1987].
2. The control system should eliminate all the higher harmonics present in the input force except the excitation frequency. This is because the shaker exhibits nonlinear behaviour at large amplitudes of vibration. Even though the magnetic field of the shaker is nonlinear, it is assumed to be linear for small amplitudes of vibration of the armature. However, when the structure under test resonates with large relative displacements, then the armature movement is in the nonlinear regime of the magnetic field and higher harmonics will be present in the input forces [Tomlinson 1987; Han 1995]. When higher harmonics are present in the forcing function, then the measured FRFs obtained could result in problems when used in analytical procedures [Tomlinson 1987]. Indeed, all the theoretical work concerning the harmonic forced response of nonlinear systems assumes a simple harmonic excitation with a constant force level as an input force. Hence, accurate tuning of the external force is essential to ensure a constant force with pure and known harmonic excitation.
3. The control system should monitor the maximum amplitude of the applied forces close to resonance conditions so that excess motion does not damage the structure. This is because when one attempts to tune the applied forces close to a resonance condition, any small deviation in the measurements and, consequently, the applied force, may result in excess motion, and thereby damage the measurement equipment or the structure. Hence, the need for a robust tuning strategy to adjust the external forces in order to ensure that they will not exceed an allowable limit is obvious.

With these considerations in mind, a control algorithm was developed in Labview software. This control algorithm uses National Instrument cards NI4472 and PCI6711 to control the force exerted by the shaker on the structure at different input frequencies. Figure 3 shows the working of the control algorithm.

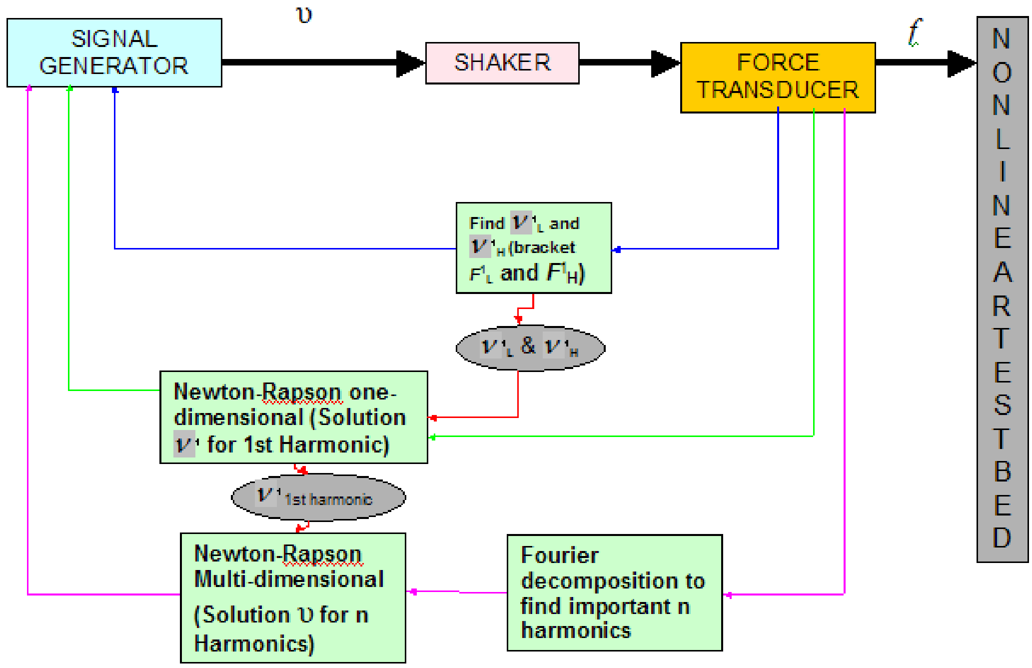



**Figure 3 Force control algorithm for obtaining FRF at large amplitudes**

Initially, a sinusoidal input signal with an initial voltage is sent from the signal generator to the shaker so that the shaker can produce a periodic force, ***f***, with a specified frequency, $\omega$, This applied force ***f*** is measured and the main harmonic component of the force, $F^1$, is extracted and compared with the force required, $f_d$. If the force, $F^1$, applied on the structure is less than, $f_d$, then the initial voltage, which contains only the harmonic component of the excitation, $V^1$ , is increased, and vice versa. Using this increment or decrement of the applied voltage, two force levels, one lower than $f_d$ , $F^1_L$, and the other higher than $f_d$ , $F^1_H$, , are identified. These force levels correspond to voltages $V^1_L$ and $V^1_H$ . This form of bracketing the approximate voltage which needs to be sent from the signal generator to produce the force needed helps the control considerably in finding the exact solution. This task is performed first by the control algorithm, and the control path is highlighted in blue in Figure 3.

Once the solution is bracketed, then the two voltages, $V^1_L$ and $V^1_H$, are used to identify the exact $V^1$ which will produce the force, $f_d$. Several classical procedures can be used to find the exact solution, $f_d$ . In this scenario, a hybrid of the one-dimensional Newton-Raphson and the bisection methods is used. The bisection method is one that cannot fail as long as the solution is bracketed. The problem with the bisection method is slow convergence. The Newton-Raphson method has poor global convergence, but it can locate the solution precisely following a good initial guess. A fail-safe algorithm is the one that combines the bisection and the Newton-Raphson methods. The hybrid algorithm takes a bisection step whenever Newton-Raphson would take a solution outside the brackets, or whenever Newton-Raphson is not reducing the size of the brackets rapidly enough.

It should be noted that the above one-dimensional nonlinear solution finds the initial voltage $V^1$, which will produce force $F^1$ that is equivalent to $f_d$ . All the higher harmonics of $f_d$ are ignored in this second stage of the control algorithm, whose path is shown in green in Figure 3. It is also worth pointing out that for the first harmonic, only the amplitude is controlled, whereas for the remaining harmonics, both the amplitude and phase must be controlled.

After obtaining a solution for the first harmonic voltage, $V^1$, this is used as an initial guess for the multi-dimension Newton-Raphson method, now using all the nonlinear equations. Before applying the multi-dimension Newton-Raphson method, a check is carried out using Fourier decomposition to evaluate the n important harmonics present in the force, $\boldsymbol{f}$, that should be controlled. When the solution is found by assuming the *n* harmonics, new harmonics may now need to be controlled since the structure is nonlinear. A new check is performed to assess weather the new harmonics have become relevant. If it is found that some new harmonics should be controlled, they are added to the previous set and the Newton-Raphson procedure is repeated. This control path is highlighted in purple in Figure 3.

Once the steady state with force, $\boldsymbol{f}_{\mathrm{d}}$ , has been achieved, the response of the structure is measured at that frequency and the frequency is incremented. The solution for the previous voltage, $\upsilon_d$ , is now used as the initial condition to find the new bracketed interval of stage 1 of the control algorithm. Likewise, the above process can be repeated for all the measurement frequencies.

Similar to controlling the force exerted by the shaker on the structure at different input frequencies to extract the equivalent linear behaviour of the structure, a constant response level control algorithm is also be developed in Labview software. The response control algorithm only exploits the first harmonic of the response when the structure is vibrating in steady state and uses the one-dimensional Newton-Raphson method to find the corresponding voltage (force) that needs to be applied to the shaker such that the first-order response (displacement, velocity or acceleration) is kept constant at certain levels throughout the different frequencies of excitation.

# 3 Controlled-Level Vibration Test on an Aero-Engine Casing Structure

The casing assembly chosen for the CLV testing is shown in Figure 4. There are two different types of joints, i.e. bolt joints and bearing joints, present in the gas turbine engine, as shown in Figure 2. This CLV test is performed to characterise bolt joints in the structure and as such, the rotor is removed from the actual assembly configuration of the engine. By removing the rotor, we are able to avoid the contribution of nonlinear behaviour from the bearings and to focus on obtaining test data to validate the bolted joints.

The structure is suspended using two elastic bungees at the middle of the casing assembly and one at the end of casing assembly, as shown in Figure 4. The suspension position and the bungees are chosen such that the first rigid body modes lie far away from the flexible modes (bending or diameteral). This allows the suspensions not to interfere with the flexible modes.

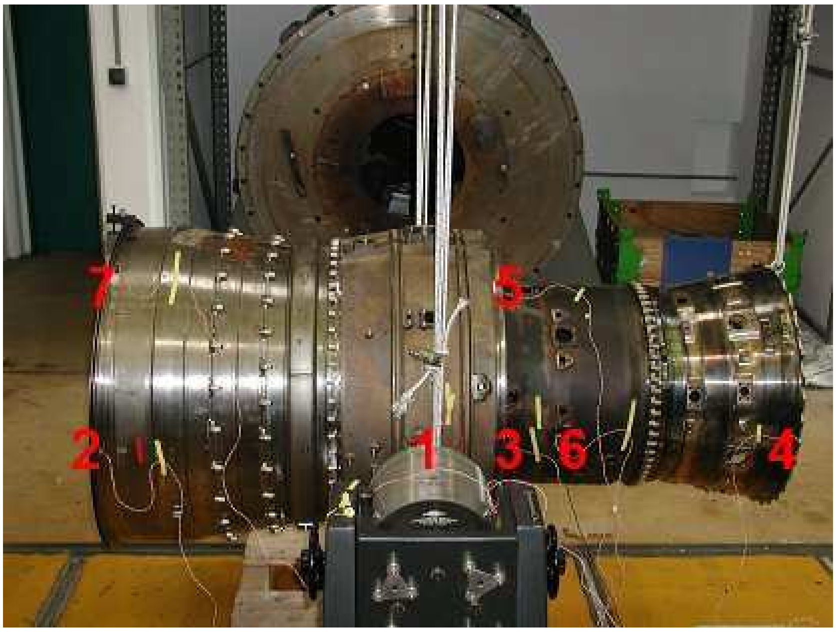



**Figure 4 Test configuration of the aero engine assembly**

Before performing the force control tests at large amplitudes, a linear hammer test was performed with a roving hammer to identify the linear modal parameters of the structure and also to find out the nodal positions of the modes. This task was important due to the time-consuming nature of the CLV testing using step sine excitations and therefore, it can only be applied for very few modes in a very confined frequency range close to the resonance.

After characterising the modes as bending, diameteral, etc., from the linear hammer test, it was decided to test the structure at frequencies close to the first horizontal bending mode for nonlinearity. One of the pairs of the bending modes, the horizontal one, is shown in Figure 5. The bending mode was chosen for the CLV test since this will activate all the bolted joints between the casings of the structure.

A 400N shaker was attached at the middle of the casing assembly to excite the structure, as shown in Figure 5. This position is chosen for the shaker since it is an anti-nodal position for the casing assembly's bending mode, and it is found to be a very rigid position to attach the shaker so that it can disseminate the large force levels to the structure without damaging the structure. Other anti-nodal positions for the bending mode lie at the two extreme ends of the casing assembly. These were not chosen since it was feared that if a high force level was applied to these component ends, there is likely to be local deformation. This will affect the distribution of energy at large amplitudes to the global modes.

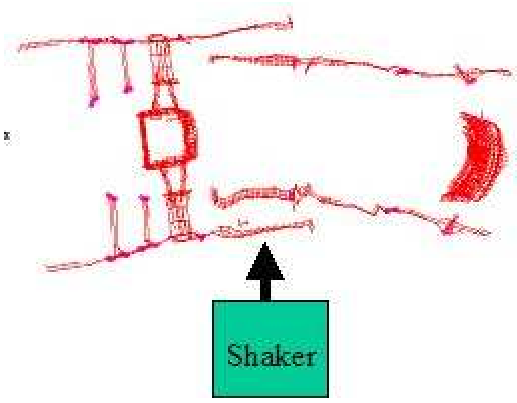



**Figure 5 Bending mode from the top**

To capture the horizontal bending profile, four transducers (1, 2, 3, 4) were placed in a horizontal line in the radial direction. Transducer 6 was also placed in the horizontal line in the axial direction. Transducers 5 and 7 were placed in vertical and near-vertical positions, facing down in the radial direction. These transducer positions are shown in Figure 4. Since the frequencies of the normal modes of the structure vary with the amplitude of vibration in CLV testing, transducers 5 and 7 are quite important to monitor the modes close to the bending mode.

Since each iteration of the Newton-Raphson process takes a considerable amount of time to converge to the correct force level and as the nonlinear behaviour at resonance needs to be captured at high frequency resolution, the measurements were made in a limited frequency range. A frequency resolution of 0.05Hz was chosen for these measurements. A fine frequency resolution is necessary in CLV testing to capture the maximum amount of information at the resonance peak due to the "jump" phenomenon observed [Stanbridge *et al.* 2001] when keeping the force level constant in CLV testing.

Only the information from the first-order harmonic will be extracted from steady-state time response for model validation. This is because higher-order frequency responses, even if detected from the measurement, are difficult to interpret and even compute from a structural model of the gas turbine engine for model validation. The first-order frequency response functions are an extension of the frequency response functions of linear structures to nonlinear structures. Similar to the measurement of FRFs of a linear structure, in the case of pure sinusoidal excitation, the first-order frequency response function of a nonlinear structure is defined as the spectral ratio of the response, $x_j$, and the force, $f_k$, at the frequency of excitation, $\omega$ ,written as:

$$H_{jk}^{11}(\omega) = \frac{X_j^1}{F_k^1} \qquad (1)$$

$H^{11}$ is the frequency response function of the fundamental frequency component of the response and the excitation. In this case, only the fundamental frequency component of the $X^1$ and $F^1$ of the response is retained, and all the subharmonics, superharmonics and combinations of both are ignored. This is similar to a linear system. In complex structures such as an engine casing, where most nonlinearities come from joints, the predominant friction behaviour of the bolted joints can be sufficiently characterised using the first-order frequency response function in a large number of cases.

### 3.1 First-Order FRFs at Constant Force Levels

The acquired first-order FRFs from the casing structure are shown in Figure 6, and their corresponding Nyquist plots are shown in Figure 7. These FRFs are obtained at Position 1 in Figure 4, at the middle of the casing assembly where the control force is applied.

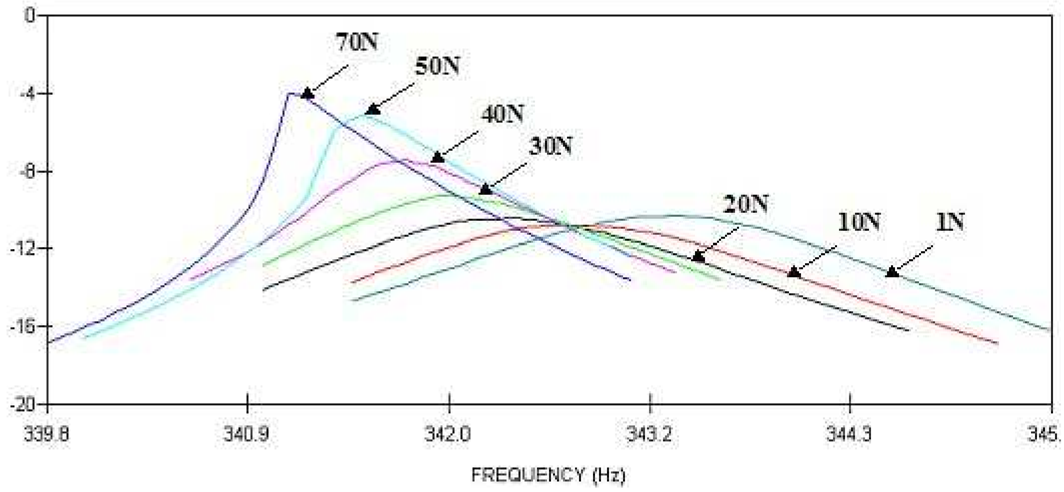



**Figure 6 First-order FRF at constant force levels**

It can be seen that when the force in increased from 1N to 70N, the resonance peak of the horizontal bending mode moves to a lower frequency, and the amplitude of the response initially decreases and then increases. This is due to the increase and decrease of friction of flange-to-flange friction at the bolted joints. The friction initially increases with the increase in amplitude (in this case, the force levels) of vibration. At high amplitudes (or high force levels), the casing bolted joints open and close and the surface area for flange-to-flange friction at the bolted joints reduces at these positions and, hence, the overall damping for the bending mode also falls at high amplitudes.

A lot more information can be obtained from the Nyquist plot from the CLV testing than from a bode plot. It can be seen from the spacing of the points of steady state measurements in the Nyquist plots in Figure 7 that the structure changes from linear to nonlinear when the force level is increased. Also, at high force levels, in this case 70N, the information at the peak of the structural resonance is only represented by very few measurements. The "jump" phenomenon, discussed earlier, can be seen from the 70N curve in the Nyquist plot where the measurement path suddenly changes its course due to instability and jumps to the next stable position available. Also, the profile of the curve distorts from a near-circle to an elliptical profile.

Due to these reasons, a Nyquist plot is a necessary tool to have in CLV testing. This will allow the test engineer to decide on the frequency range and interval of measurement to capture the nonlinear behaviour of the structure. These decisions are crucial in CLV testing since each measurement point in the Nyquist plot, shown in Figure 7, takes a considerable amount of time to acquire due to the optimisation process discussed in Section 2.

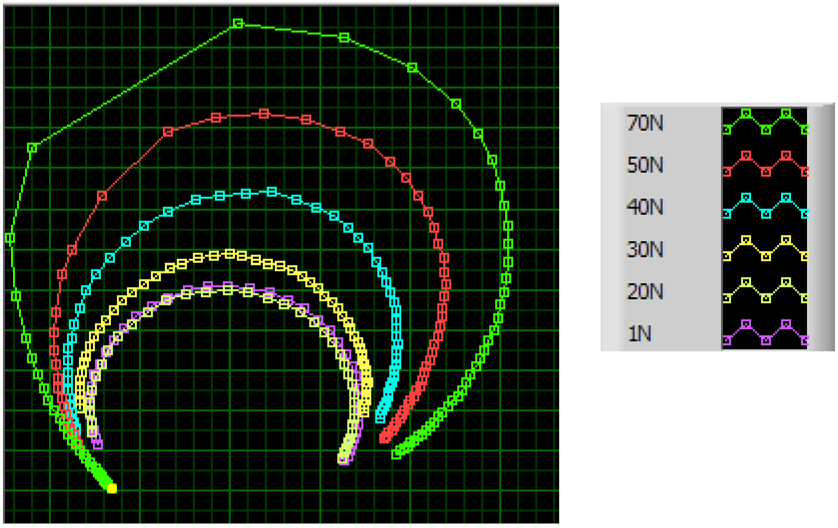



**Figure 7 Nyquist plot of the first-order FRFs at force levels**

From the experienced gained from testing the casing assembly, the limitations related to using CLV testing for nonlinear model validation can be described, as follows:

(i). The most information from the CLV testing for nonlinear model validation, i.e. the variation of modal parameters with the amplitude of vibration, is obtained at the resonance peaks, where the highest amplitude occurs. However, due to the jump phenomenon, limited information can only be obtained from these resonance peaks for model validation. Stanbridge *et al.* [2001] have described a method to obtain more information from the overhang part of the FRF, avoiding the jump phenomenon.

(ii). The data obtained from CLV testing at the resonance peak also depends on the frequency resolution of the test. The smaller the frequency resolution, the greater the number of measurements that can be obtained from the test to characterise the structure's nonlinear behaviour. However, there is a limit to the minimum frequency resolution one can use in CLV testing due to the constraint of the signal generator.

(iii). For a practical engineering structure, there is a limit to the force level applied on the structure without damaging the structure or the measurement equipment. Hence, the nonlinear behaviour of the structure can only be characterised until this limit.

## 3.2 First-Order FRFs at Constant Response Levels

Unlike the constant force level first-order FRFs, the constant response level first-order FRFs look like the FRFs obtained from a linear system to a certain extent. This is because most of the nonlinear features, such as joints, exhibit response amplitude-dependent equivalent linear behaviour. Hence, standard modal analysis techniques can be used to obtain the linearised characteristics of the system at different response amplitudes. The first-order FRFs obtained at constant response amplitude level are plotted in Figure 8, and their corresponding Nyquist format is plotted in Figure 9.

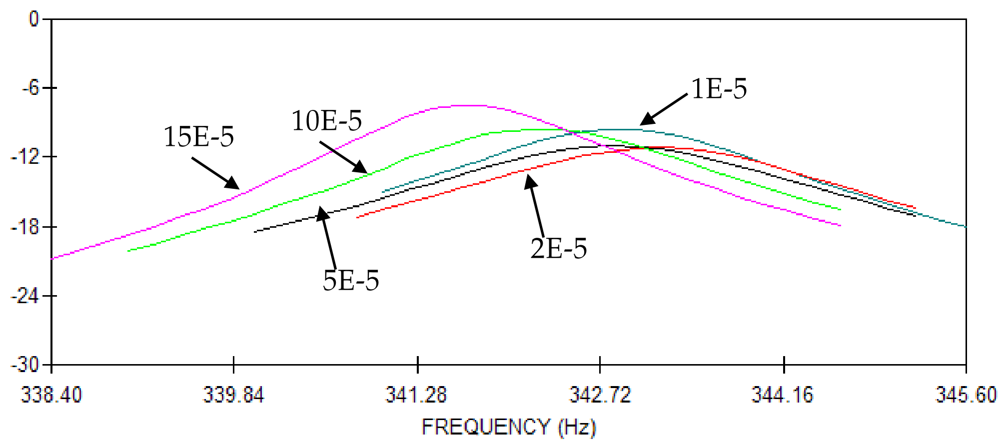



**Figure 8 First-order FRFs from CLV testing at constant response amplitude levels (response amplitude defined in meters)**

From the spacing of points in the Nyquist plot of Figure 9, the FRFs obtained at constant response amplitude level exhibit equivalent linear behaviour of the casing assembly structure at different frequencies.

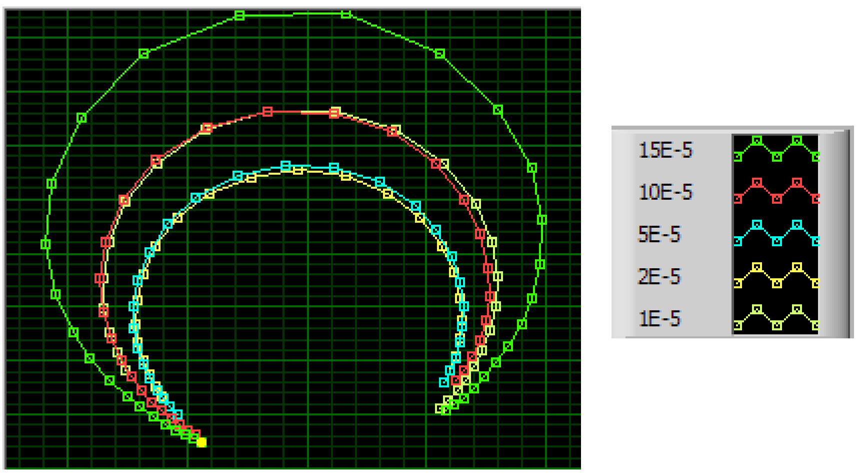
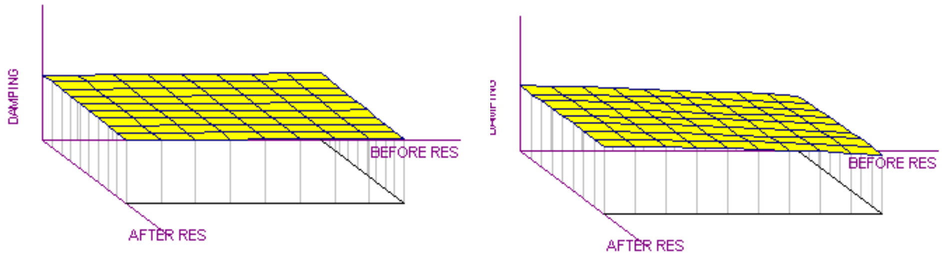



**Figure 9 Nyquist plot of the first-order FRFs at constant response amplitude levels**

The damping carpet plot [Ewins 2000], shown in Figure 10, demonstrates no sign of major nonlinearity in the test data obtained by keeping the response amplitude level constant in the CLV test.




Amplitude Level = $1x10^{-5}$m Amplitude Level = $1.5x10^{-4}$m

**Figure 10 Damping carpet plot of the first-order FRFs at constant response amplitude levels**

Hence, it can be deduced that the equivalent linear behaviour of the engine casing assembly can be obtained from constant response amplitude CLV testing. Furthermore, standard modal analysis techniques can be used to obtain the equivalent linearised modal parameters of the structure.

## 4 Nonlinear Characterisation for Incorporating the Behaviour into Design Models

In the case of linear structures, the FRFs are the same regardless of the initial condition or input signatures. However, in structures with nonlinear behaviour, the first-order FRFs vary for different input levels. The degree to which the first-order FRFs vary for different input levels can be used to characterise nonlinear systems. For nonlinear systems, the mass, stiffness and damping parameters vary with the amplitude of vibration and, hence, linear methods cannot anymore be used to extract the modal parameters of the system.

A method which extends linear modal analysis to nonlinear systems was first introduced by He and Ewins [1987]. Lin, Ewins and Lim [1993] further extended this method to extract modal parameters from complex modes. This method is used to extract and characterise the nonlinear modal parameters from first-order FRFs is applied to the data obtained from force-control CLV tests on the engine casing assembly. The variations of natural frequency with response amplitude and of damping with response amplitude are plotted in Figures 11 and 12, respectively.

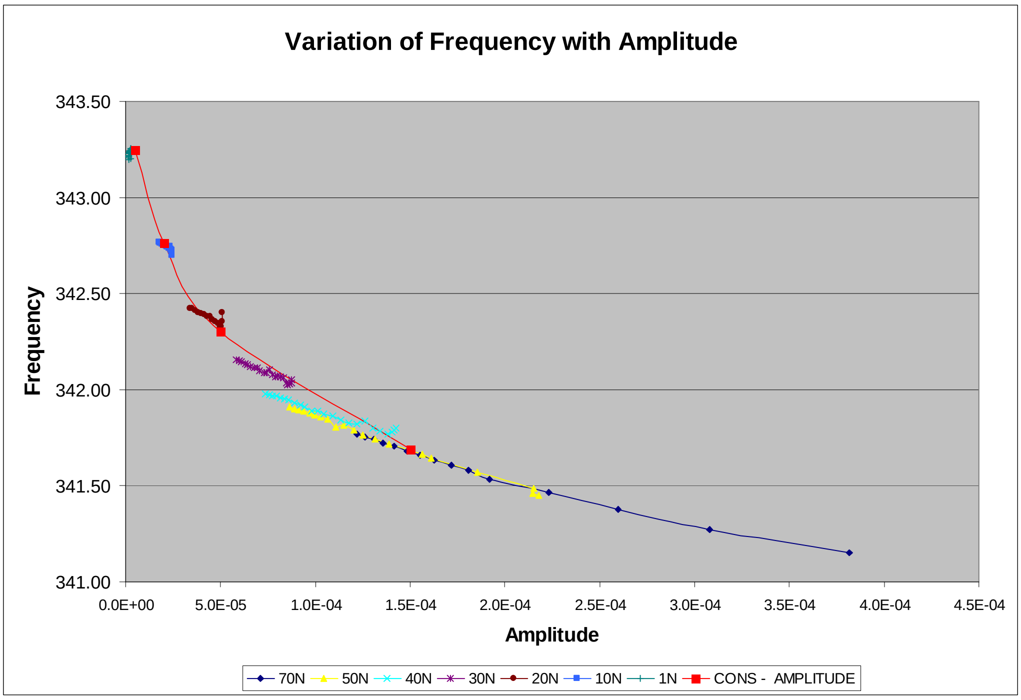



**Figure 11 Variation of natural frequency with displacement amplitude**

Also, in Figures 11 and 12, the nonlinear modal parameters obtained from linear modal analysis on the first-order FRFs at constant amplitude response levels are plotted in red. The variation of natural frequencies obtained from the constant-amplitude CLV test correlated well with the constant-force CLV test. However, the damping correlates well for lower amplitudes but deviates for higher amplitudes of vibration.

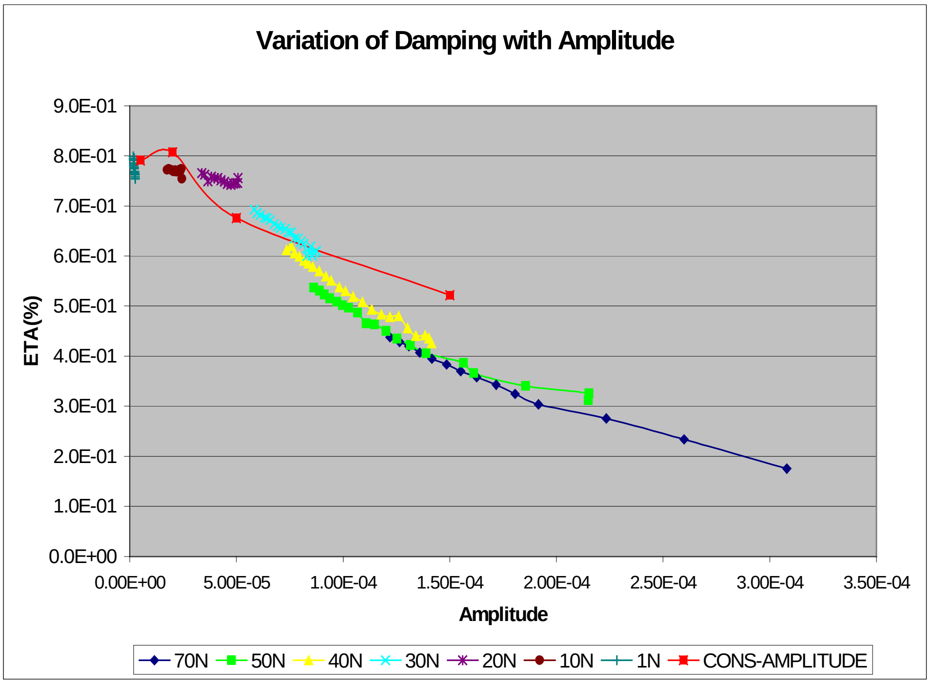



**Figure 12 Variation of damping with displacement amplitude**

The characteristics of friction between bolted flanges, discussed in Section 3.1, are clearly visible from the natural frequency and damping variation plot. From the plot, initially, at lower amplitudes of vibration, the friction, or, in this case, the damping, increases and after a critical level, decreases due to the opening of bolted joints. Also, due this effect, the natural frequency also reduces exhibiting a softening effect. The visibility of these characteristics from these plots, i.e. the variation of stiffness and damping with amplitude of vibration, can be used to choose appropriate nonlinear elements, i.e. Coulomb friction, bilinear stiffness, polynomial damping, etc., for bolted joints such that this nonlinear element can be incorporated into the design (linear) FE model and updated using the first-order FRF measured at different positions on the casing assembly [Boeswald *et al.* 2002].

# 5 Conclusions

In this research work, controlled-level vibration tests are used to characterise the local nonlinear behaviour of bolted joints in an engine casing assembly. In doing so, a constant force and constant response control algorithm were developed in LabView software and applied to a specific mode, in this case, the bending mode, of the engine casing assembly. First-order FRFs were acquired using both constant-force and constant-amplitude CLV testing. The information in the first-order FRFs is discussed from both bode and Nyquist formats. Also, the importance of displaying information from the first-order FRFs in Nyquist format for selecting the frequency range and frequency interval of measurement for CLV testing is discussed. Some of the limitations of CLV testing are revealed in this chapter. Once the first-order FRFs have been acquired from the structure, an extraction process developed by Lin, Ewins and Lim [1993] is used to extract the nonlinear modal parameters from the first-order FRFs. The information from this extraction process, i.e. the variation of natural frequency and damping with vibration amplitude, can be used to select an appropriate nonlinear element for the interfaces to update the structural model for nonlinear behaviour.

## 6 Acknowledgement

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