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Rotordynamics and Uncertainty of Variables in Gas Turbine

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0 Abstract

In future gas turbine designs the complexity of rotor dynamic simulation for design assessment will increase in two directions, firstly more detailed physics simulation, and secondly, a broader system scope. The increasing pressure to do more sophisticated simulations at reduced cost and the need especially during the preliminary design phase to get quick answers with reduced effort makes it necessary to assess the robustness of the design with respect to e.g. material process control, manufacturing tolerance, machining, assembly, or in-service conditions.
A stochastic simulation process has been developed and applied to a gas turbine design. For several key design parameters sensitivity data of rotor dynamic behaviour have been determined. Some examples of application of these results in the design process are presented.
This stochastic rotordynamic simulation philosophy has several benefits:

- during the preliminary design phase it enables with low effort a quick evaluation of alternative design options
- enables to value the robustness of the design e.g. manufacturing tolerance and material properties
- risk reduction of late design changes
- increase of engine reliability
- Improve in-service robustness
- Avoid unplanned costs

1 Introduction

Probabilistic thinking was a mid-17th century artefact originating in a famous correspondence between Fermat and Pascal -- a correspondence on which Huygens based a widely read textbook: "On Calculating in Games of Luck" (1657).

Modern turbines of aero engines are high technology products, which are characterised by high performance, high reliability, low total lifecycle cost and low emissions. Further progress in the development of gas turbines requires both continuous optimisation on component and system level as well as the use of new and innovative technologies. Thus the design is often pushed closer to the physical limits. This demands an excellent understanding and predictability of the structural behaviour.

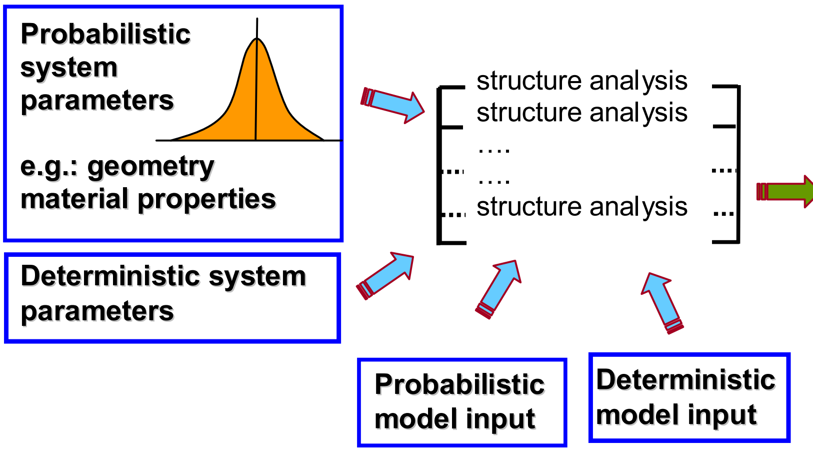

Figure 1: Probabilistic model description

Due to the considerable costs associated with real-life testing of gas turbines, the knowledge on structural behaviour and failure mechanisms is often gained from validated numerical models. However, in order to obtain reasonable results of the simulation models, the reality has to be taken into account as close as possible. One important part of reality is the uncertainty in the design, the material properties as well as the loading and operation conditions. Therefore, for robust performance under all possible conditions the scatter of important variables has to be taken into account right from the beginning of the development process.

Probabilistic design methods were developed to consider the intrinsic design scatter. In contrary to deterministic analysis approaches, the most important design parameters are now being defined by probability distribution functions rather than by deterministic values. As a result, the critical speed or the probability of failure of the stochastic variables is evaluated.

Although the principles of modern probabilistic methods have already been developed since the beginning of the last century, their application to industrial practice is only of recent date. Reasons for this late use are the enormous computational effort since the simulation of probability typically bases on many deterministic analyses. However this disadvantage is minimised due to the rapid development of the computer technique in the last decades, the more efficient numerical techniques of the continuum-mechanical models and advances in the probability methods themselves.

Pioneering work for the application of probabilistic methods has been achieved by the automobile and aircraft industry. As an example, since the middle 1990's probabilistic design methods have been successfully used in crash simulation of cars to improve the error-tolerance and robustness of the design [1], more general [9].

All structural systems operate in random environments. Moreover, the structural systems themselves imply a certain degree of uncertainty. As a consequence, the real-live response of the structure scatters and, dependent on the robustness of the system, this scatter may have significant influence on functionality and reliability. Contrary to this, standard mechanical analysis is based on deterministic models, which relate a specific loading and a specific realisation of the structural system (boundary conditions, material properties, geometry) to the structural response. Clearly, the deterministic approach does not take into account the inherent uncertainty, which is characteristic for each real engineering system and therefore an important part of physics is not considered in deterministic simulation models. This behaviour might be described by a probabilistic design approaches, figure 1.

2 Deterministic and Stochastic Variables

Nearly every parameters of the analytical and/or numerical model imply a degree of uncertainty. The extent of uncertainty depends on the level of physics involved in the model and also which problem is studied in detail. However, in many practical applications only a few variables are dominant, which influence the scatter of the structural behaviour most.

In general there are several systems to categorise probabilistic variables, [2, 3]. Considering the physical aspects of design analysis, one can distinguish:

- *thermal and structural loads (temperature, heat transfer coefficient, pressure, etc.)*
- *boundary and initial conditions (structural clamping, contact conditions, etc.)*
- *material properties (elastic modules, yield strength, etc.)*
- *geometry (manufacturing tolerances, assembly tolerances, etc.).*

Moreover, the scatter of probabilistic variables might be categorized in:

- *scatter that can be reduced by improving the knowledge about the system and the input data (insufficient data base, incomplete understanding of model, etc.)*
- *scatter that is due to the inherent randomness of the physical process (wind load, etc.). The last categorization given some hints, where to spend effort to improve the simulation.*

3 Description[10]

A deterministic model can be described using the general relation

$$\mathbf{w} = \Phi(\mathbf{v})$$

where **v** denotes the deterministic parameters of the input and the model whereas **w** is the deterministic structural response.

A closed analytical function that relates **v** to y can only be found for simple geometry and load conditions. For complex structures, such as a gas turbine, numerical methods (e.g. finite element methods or finite difference methods) are applied to assess the structural

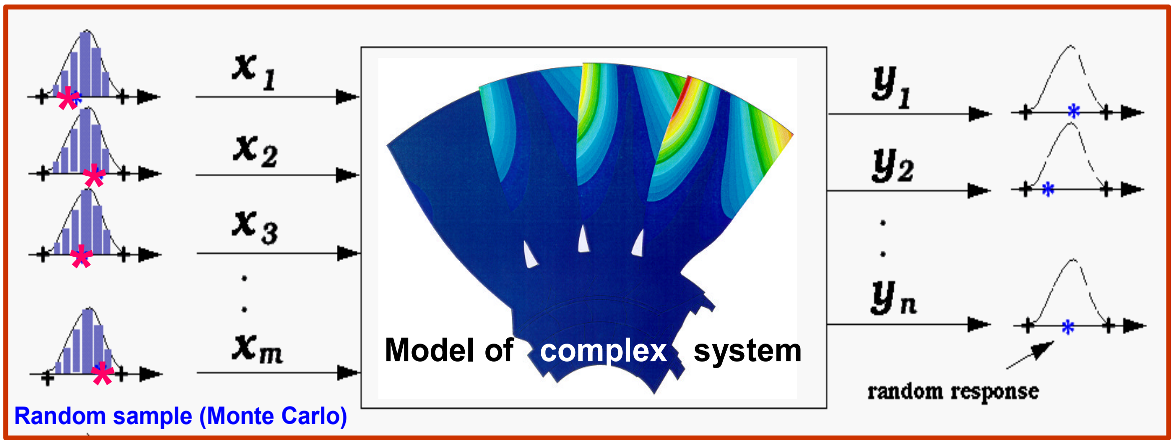

Figure 2: formalism

behaviour for a specific set of **v** deterministic parameters.

In a probabilistic analysis the dominant deterministic parameters are replaced by stochastic variables

$$\mathbf{x} = (x_1, x_2, ..., x_n)$$

where each realization of **x** corresponds to a different set of model input data and model parameters. These sets of variables **x** are characterized by appropriate probability distribution functions of Gaussian and/or non-Gaussian type. It is necessary, however, that the stochastic variables $\mathbf{x}_i$ are independent from each other. In case of correlated variables, a transformation into uncorrelated basic variables has to be performed first.

Amongst them are the **R**esponse **S**urface **M**ethods (RSM) and the **M**onte-**C**arlo **S**imulation (MCS) procedures. These two methods shall be introduced briefly in the following two sections.

3.1 Monte-Carlo Simulation

The first step of a **M**onte-**C**arlo **S**imulation (MCS) is a random number generation which gives single value of the stochastic variables **x** corresponding to an user-defined probability distribution function. There are different methods to generate the realisation of the stochastic variables, such as descriptive sampling or the **L**atin **H**ypercube **S**ampling (LHS), which is applied here. The second step of the MCS is the structural analysis for each set of realisations with the numerical code, figure 2. These runs are independent deterministic simulations and can be processed simultaneously. The third step is the statistical evaluation of the output variables.

Furthermore, the statistic interpretation of the stochastic input and output variables gives important indications with respect to robustness of the system and can indicate outliers. One advantage of the MCS to other probabilistic methods is that the precision of the results is independent of the number of stochastic variables.

3.2 Response Surface Method

In contrary to MCS, in the Response Surface Method the deterministic calculations are carried out systematically according to a test plan. The results of the deterministic analyses are then used to determine an approximate failure function g(b, y). The centre of the test plan should be located nearby the failure criterion g(b, y)=0. Therefore, an iterative procedure is required. There are several possibilities to describe the approximate failure function. For details see [8]. In [8] also the quadratic and cubic formula of the response surface method are given.

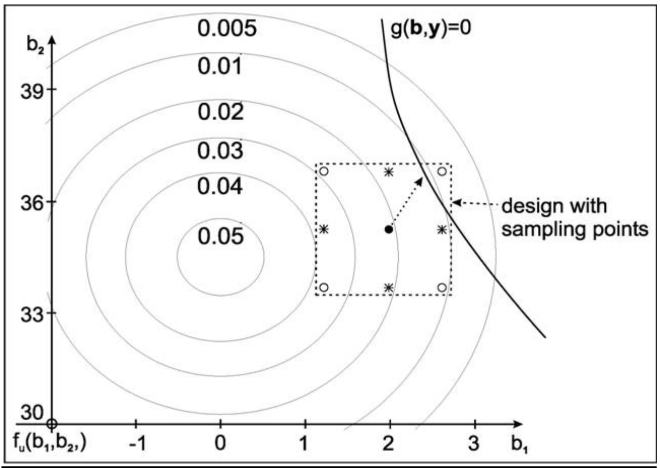

Figure 3: Response Surface Method [10]

4 Application

Figure 4 shows the simplified model of an twin shaft engine with five bearings. This example will be used to study the influence of different parameters stochastically distributed to the critical speed resonance. These parameters can be for example: the mass of the different engine modules, the bearing stiffness, and the distance between the two bearings.

The calculation is done using iSight to run the task plan and to provide the random choice of the stochastic variables in each run, see figure 5 for the task plan and figure 6 for a possible distribution used for the bearing stiffness (here a lognormal distribution is used, also useful normal, exponential or sometimes uniform distributions). The mass variation (as shown in fig. 4) is assumed to be normal distributed, whereas the bearing distance variation is uniform distributed.

The numerical calculation tool is a so called matrix transfer method, for details see e. g. [4]. With the aid of VBA [5] the output-file of iSight [6] is formatted for the use with Excel. For the manipulation of the data principles from [7] are used.

Starting with zero rotation every 1000 rpm a (probabilistic) calculation is done. At each time step at least 500 calculations with randomly generated values of the stochastic variables are carried out. At the end of each tome step calculation the required information as speed and critical speed are written to an output file.

The pure computing time is about 50 hours using using a LINUX-workstation with 4 GByte memory and 3.2 GHz Intel processor.

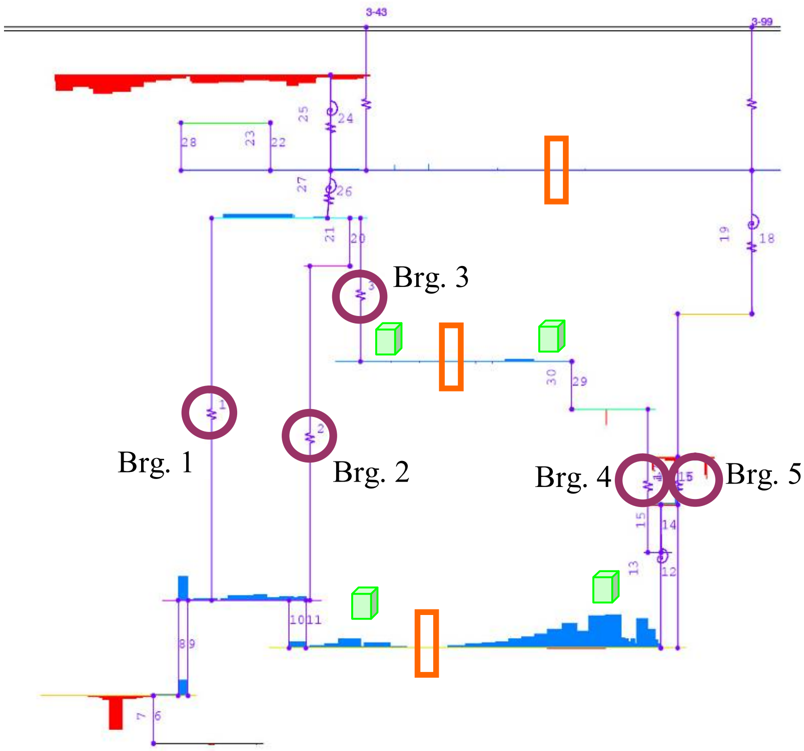

Figure 4: Variation of bearing distance (schematic diagram of casing, low and high pressure system of a gas turbine, only to show at which location changes were made)

change the length of rotor 1, 2 and casing (same magnitude)

change stiffness of bearings 1 to 5, single or combinations or all

add mass at predefined locations on the rotor 1 and/or 2

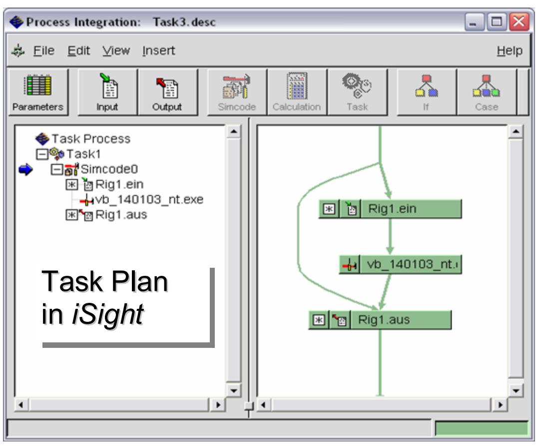

Figure 5: task plan in iSight

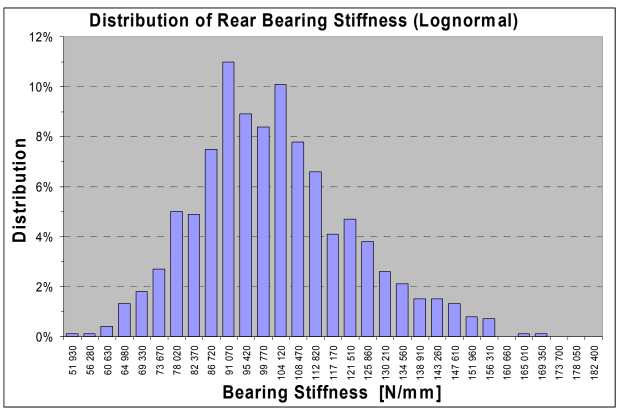

Figure 6: lognormal distribution of bearing stiffness

One severe problem during the calculations occurs. The amount of critical speeds in the predefined range from 0 to 20.000 rpm may vary at each chosen time step. This is due to the scatter of the stochastic variables. Therefore the interpretation of the result is at the moment very time consuming.

5 Results

This study is in the beginning. Therefore only some results exist.

Figure 7 shows the Campbell diagram of the critical speeds. The model in figure 4 was used. The black lines starting at zero represent the first engine order of the low pressure system and high pressure system.

The lines in the different colours represent critical speeds as calculated with nominal values of the variables. The markers at every time step represents the variation of the critical speed in dependence from the random choice of the bearing stiffness. A normal distribution was assumed and the following stiffness values are used.

stiff-ness	mean value [N/mm]	std deviation [N/mm]
brg 1	400.000	38.000
brg 2	300.000	29.000
brg 3	400.000	38.000
brg 4	440.000	40.000
brg 5	340.000	32.000

Table 1: mean bearing stiffness values and standard deviation

For the first seven critical speed the influence of stiffness variation within the above limits shows nearly no influence. For higher frequencies the influence of the variation has a bigger effect which is seen in the scatter of the values at each time step.. This is importing for higher engine orders which may be excited then at different engine speed.

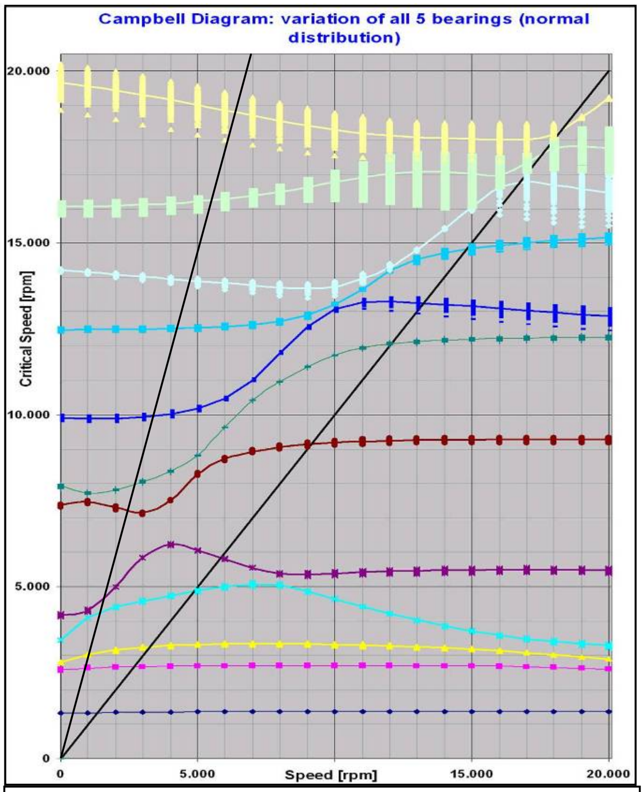

Figure 7: Campbell diagram: variation of bearing stiffness (normal distribution)

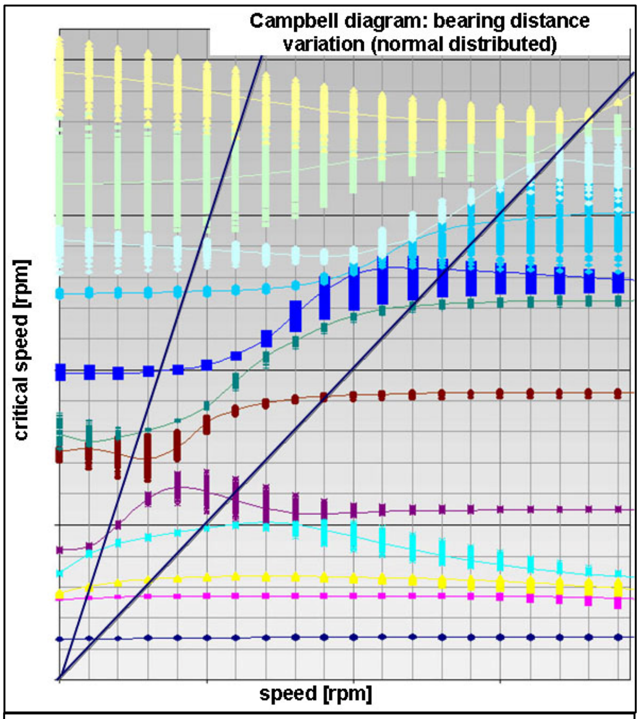

Figure 8:Campbell diagram: variation of bearing distance (normal distribution)

The same calculation was done as shown in figure 8 now with variation of the distance between the front and rear bearing system. For each system (casing, low pressure and high pressure system) the same value of the distance variation was used (see figure 4).

distance variation	mean value [mm]	std deviation [mm]
length	10	2,75

Table 2: mean distance value and standard deviation

The distance was changed simultaneously in the casing, low pressure and high pressure system. In this case the influence of variation is much bigger than in the example before. The scatter of the higher critical speed increases.

The next step will be mass variation at sensitive locations of the rotor system. After that combinations of stiffness and distance or stiffness and mass or distance and mass are investigated and checked against the singular results. At the end all a combination of the three stochastic variables will be done.

6 Summary

A stochastic simulation process has been developed and applied to a gas turbine design. For several key design parameters sensitivity data of rotor dynamic behaviour have been determined and will be extended in future. Some examples of application of these results in the design process are presented.
This stochastic rotordynamic simulation philosophy has several benefits:

- during the preliminary design phase it enables with low effort a quick evaluation of alternative design options
- enables to value the robustness of the design e.g. manufacturing tolerance and material properties
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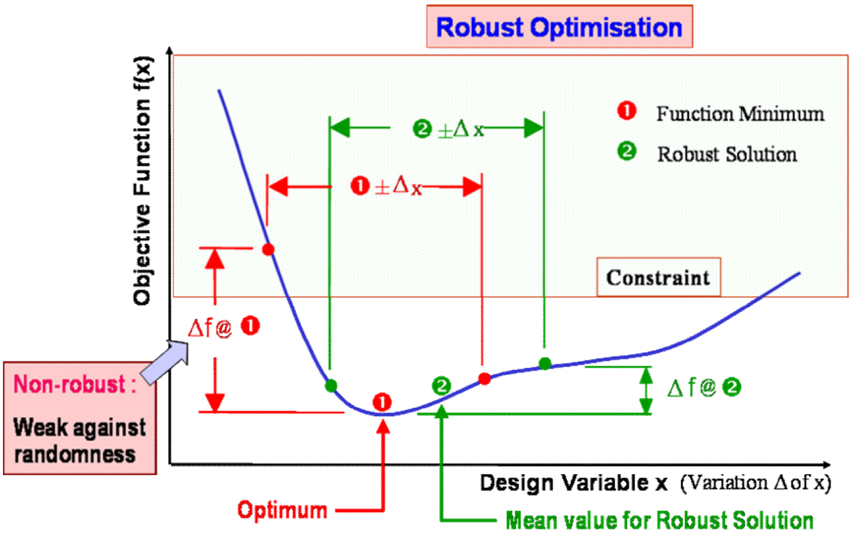
7 Outlook

Despite the additional numerical effort probabilistic methods are very valuable. In contrary to a single deterministic result, probabilistic methods describe system behaviour for the whole range of realisations and operation conditions, which gives information about

- the robustness of the structure,
- the probability of failure and
- the sensitivity of the stochastic variables.

As a result, additive conservative assumptions in each of the design phase can be avoided. Moreover probabilistic methods support risk assessment procedures and therefore contribute to a lifetime extension of components if possible. This leads to a robust optimisation of the rotordynamic model in an early design phase.



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