

# Achieving maximum thermal efficiency with the simple gas turbine cycle

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**Abstract.** The thermal efficiency of the ideal Joule cycle operating on a perfect gas is only a function of pressure ratio and the isentropic exponent of the gas. When component efficiencies are lower than 100%, then thermal efficiency becomes also a function of burner exit temperature. Calculations for a perfect gas yield that the achievable thermal efficiency increases monotonously with burner temperature in such a way that the optimum pressure ratio is dependent on the efficiency level and the burner temperature. The higher the burner temperature is the higher is also the optimum pressure ratio. However, in the real world air and combustion gases are not perfect gases and quite obviously the stoichiometric fuel-air-ratio limits the achievable burner temperature. One might now assume that the maximum thermal efficiency is achieved at or near to the stoichiometric fuel-air-ratio, however, this is not the case.

The thermal efficiency of a cycle in which all the turbo-machines have 90% polytropic efficiency and cooling air is not taken into account is maximal at a burner temperature corresponding to a fuel-air-ratio which is not higher than 50% of the stoichiometric value and independent from the fuel composition. If cooling air is modeled then the location of the maximum thermal efficiency is at a 10% higher value. The reason why the maximum thermal efficiency happens not to be at the highest temperature is the non-linear correlation between fuel-air-ratio and temperature increase in the burner. Neither the temperature dependence of specific heat nor the water vapor content of the combustion gas are the reason for the maximum thermal efficiency existing at fuel-air-ratios lower than the stoichiometric value as reported in literature.

Since modern gas turbines employ burner temperatures not too far below the optimum temperature it must be concluded that in the future increasing burner exit temperature is not a way to increase thermal efficiency as it was in the past. Increasing pressure ratio yields a moderate improvement potential and true improvements in thermal efficiency are only possible with alternate gas turbine configurations.

## 1 INTRODUCTION

One may ask: what is the relevance of the simple cycle for the future of the aero engine gas turbine – which is typically a turbofan. The answer to this question is that the turbofan engine cycle can be easily split into two parts: There is a core stream process which comprises of the primary flow commencing with ambient conditions up to a location within the low pressure turbine which is defined in such a way that all the compressor power needed for the core stream is covered. The second and third process parts deal with the bypass stream compression and expansion as well as with generating the core stream thrust.

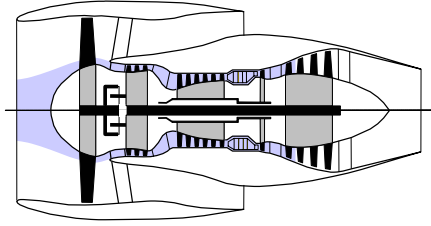


Fig. 1 Core Stream Process of a Turbofan

Apart from the compression of the incoming air from ambient conditions to inlet total pressure and temperature the core stream process is exactly the same as that of the simple cycle gas turbine.

The following discussions are restricted to the thermal efficiency of the gas turbine cycle and they do only touch shortly the specific power, i.e. the power per unit mass flow. Keep in mind that for aero propulsion applications the power developed per frontal area, per volume and per weight is an equally important attribute of any gas turbine core as its thermal efficiency.

The abstract of reference 3 begins with the statements: “Thermal efficiency of gas turbines is critically dependent on temperature at the turbine inlet; the higher this temperature, the higher the efficiency. Stoichiometric combustion would provide maximum efficiency”. This view about gas turbine efficiency is widely spread, however, it is incorrect. In reference 1 it is shown that when gas properties are modeled accurately the variation of cycle efficiency with turbine inlet temperature at constant pressure ratio exhibits a maximum at temperatures well below the stoichiometric limit. The authors of ref. 1 come to the conclusion that the dominant influence for this unexpected phenomenon comes from the change of composition of the combustion products with varying fuel-air-ratio, particularly the contribution from the water vapor.

This paper starts with the findings from ref. 1 and extends the study to effects that were not included in the referenced paper. At first the un-cooled cycle performance is discussed; the accuracy of the gas property modeling is improved in several steps with the aim of isolating the source of the efficiency maximum at temperatures well below the stoichiometric limit.

## 2 SCHOOLBOOK WISDOM

We consider the cycle of a simple gas turbine with 90% polytropic efficiency for both the compressor and the turbine and no pressure losses in other parts of the cycle. To simplify the considerations further the mass flow through compressor and turbine are assumed to be equal. With other words, the amount of high pressure air leakage is equal to the amount of fuel added.

### 2.1 Definition of Thermal Efficiency

The thermal efficiency of this cycle is equal to the specific turbine shaft power minus specific compressor shaft power divided by the amount of heat added in the burner. For constant specific heat one can write

$$\eta_{th} = \frac{H_T - H_C}{H_B} = \frac{T_4 - T_5 - T_3 + T_2}{T_4 - T_3} = 1 - \frac{T_5 - T_2}{T_4 - T_3}$$

If component efficiencies are 100% then holds

$$\frac{T_3}{T_2} = \frac{T_4}{T_5} \quad \text{respectively} \quad \frac{T_5}{T_2} = \frac{T_4}{T_3}$$

This yields for the ideal Joule cycle that its thermal efficiency is only a function of pressure ratio:

$$\eta_{th} = 1 - \frac{T_2}{T_3} = 1 - \left( \frac{P_3}{P_2} \right)^{\frac{1-\gamma}{\gamma}}$$

If the component efficiencies are not 100% and the gas properties (isentropic exponent  $\gamma$  and gas constant R) are not constant then the formula becomes

$$\eta_{th} = \frac{\frac{\gamma_T}{\gamma_T - 1} * R_T * \frac{T_4}{T_2} * \left[ 1 - \left( \frac{P_2}{P_3} \right)^{\frac{\gamma_T - 1}{\gamma_T}} \right] * \eta_T - \frac{\gamma_C}{\gamma_C - 1} * R_C * \left[ \left( \frac{P_3}{P_2} \right)^{\frac{\gamma_C - 1}{\gamma_C}} - 1 \right] / \eta_C}{\left( \frac{T_4}{T_2} - \frac{T_3}{T_2} \right) * \left( \frac{\gamma_C}{\gamma_C - 1} * R_C + \frac{\gamma_T}{\gamma_T - 1} * R_T \right) / 2}$$

Since the temperature ratio  $T_3/T_2$  is directly coupled with  $P_3/P_2$  and compressor efficiency it is obvious that the thermal efficiency of the simply cycle gas turbine is a function of pressure ratio  $P_3/P_2$ , temperature ratio  $T_4/T_2$ , component efficiencies and the properties of the gas.

## 2.2 Constant Gas Properties

The most simple model of the cycle employs constant gas properties which means in the example shown below  $\gamma_C = \gamma_T = 1.35$  and  $R_C = R_T = 287$  J/kg/K. Evaluating thermal efficiency over a wide range of pressure ratios and temperatures with this simple gas property model yields the results shown in figure 2. For a better view on the optimum thermal efficiency islands the parametric study is extended to pressure ratios well beyond any realistic case.

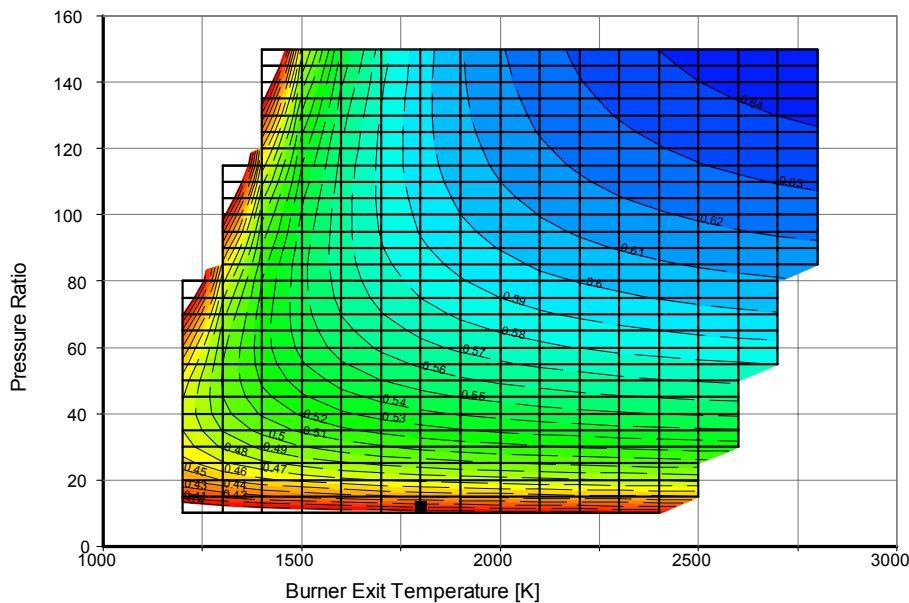


Fig. 2: Thermal Efficiency Evaluated with Constant Gas Properties

The tendency in this figure is clear: increasing burner exit temperature at constant or increasing pressure ratio yields improved thermal efficiency. However, if we limit ourselves to a realistic pressure ratio of 40, for example, then the potential gain in thermal efficiency at temperatures above 2000K is not very big. The top left corner is cut off because there the compressor exit temperature exceeds the burner exit temperature. The right border of the parametric study represents the maximum temperature achievable if Kerosene is used as fuel

### 2.3 Temperature Dependent Gas Properties

Burning hydrocarbons (Kerosene, JP4 or Diesel, for example) with air leads to combustion gases that have practically the same gas constant as dry air. Thus the assumption  $R_C = R_T = 287 \text{ J/kg/K}$  is valid, but the isentropic exponents  $\gamma_C$  and  $\gamma_T$  are in reality not constant but change significantly with temperature. Moreover, the magnitude of  $\gamma_T$  depends also from the composition of the combustion gases, i.e. the fuel-air-ratio.

The gas properties of combustion gases as well as the temperature rise due to combustion used in this paper have been calculated with the NASA CEA program, see ref. 5 and 6. The effect of pressure on the heat release is taken into account; the pressure effect on the other gas properties (isentropic exponent, gas constant, enthalpy and entropy) is neglected.

Rerunning the parametric study with variable isentropic exponents - i.e.  $\gamma_C=f(T)$  and  $\gamma_T=f(T, \text{far})$  - yields the results presented in fig.3.

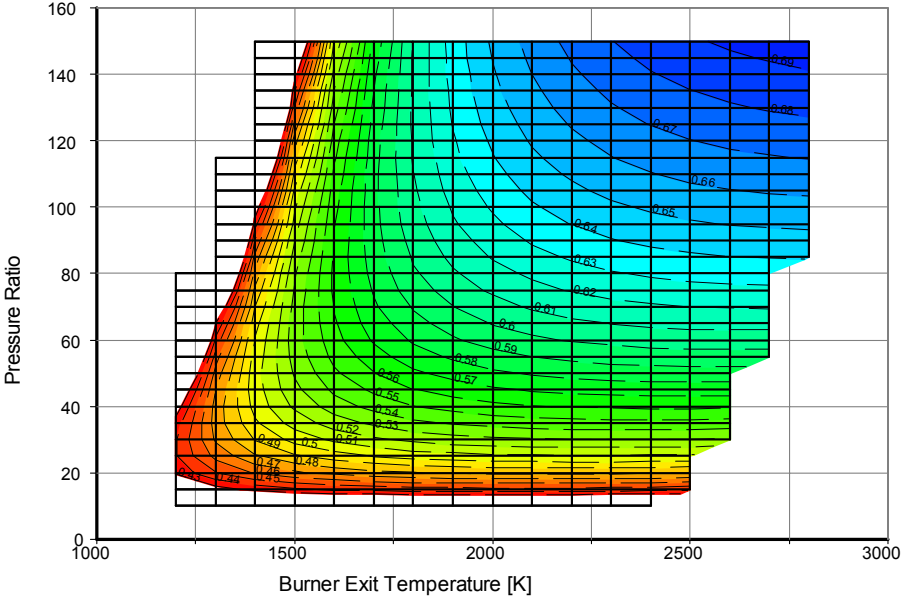
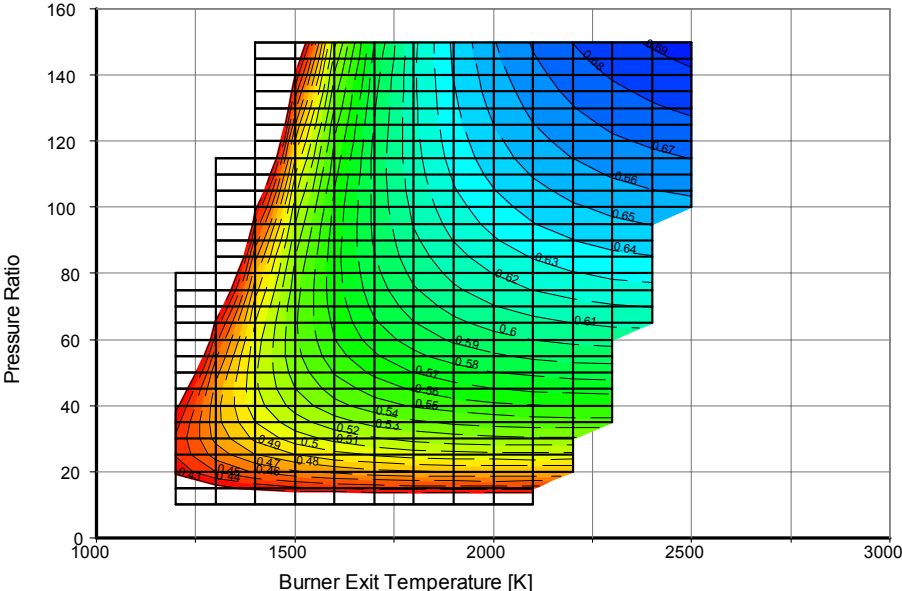


Fig. 3: Thermal Efficiency Evaluated with Temperature Dependent Gas Properties

There is not much difference between figures 2 and 3 and especially the tendency that the highest burner temperature yields the best thermal efficiency is the same in both models. The question remains if a difference in the gas constant between compressor and turbine can change the basic shape of the efficiency contour lines. In the conclusions of ref. 1 it is speculated that steam injection could have a major impact on the location of the maximum efficiency. The reasoning behind this is that the increased amount of water vapor in the combustion exhaust changes the specific heat in the expansion process significantly.

To study this effect the exercise has been repeated with steam injection (steam-fuel-ratio 1) into the burner. As can be seen from fig. 4 again no significant change in the shape of the contour lines can be observed except that the maximum temperature achievable is reduced. This comes from two effects: first the percentage of oxygen in the gas consisting of a mixture of air and steam is lower than in dry air. Second some of the heat released by the chemical reaction is needed to heat the steam to burner exit temperature.



**Fig. 4:** Thermal Efficiency with Steam Injection (Steam-Fuel-Ratio=1), Temperature Dependent Gas Properties

The authors of ref. 1 come to the conclusion that the dominant influence for the thermal efficiency maximum which they have found originates from the change of composition of the combustion products with varying fuel-air-ratio, particularly the contribution from the water vapor. They make the specific heat of water vapor – which is significantly different to that of air and other combustion products – responsible for the maximum efficiency being at temperatures lower than stoichiometric. The results shown above seem to be a contradiction to the findings from ref. 1 because no efficiency maximum below the stoichiometric temperature could be found even when the gas properties are modeled with the same accuracy as in ref. 1. Especially it has been demonstrated by the steam injection example that the gas properties of water vapor do not create an efficiency maximum at temperatures well below the stoichiometric limit.

In the calculations presented up to now only simple formulae as found in schoolbooks have been employed. Next the full blown cycle code from ref. 4 will be used for evaluating thermal efficiency.

### 3 FULL CYCLE CALCULATION

There is no difference between the gas property model in GasTurb and that employed for getting the results reported above. Actually the calculations for section 2 of this paper have been done with the same code which allows the user to add his own formulae as needed.

### 3.1 Definition of Thermal Efficiency

In the cycle code the thermal efficiency is defined as

$$\eta_{th} = \frac{H_T - H_C}{W_F * FHV}$$

The difference between this definition and the one used in section 2 is in the denominator: instead of the burner temperature difference  $T_4 - T_3$ , multiplied by the mean specific heat, here the product of fuel flow  $W_F$  and fuel heating value  $FHV$  is used. This is reasonable because in the real world one has to pay for fuel, not for a temperature difference as implied with the schoolbook definition of thermal efficiency.

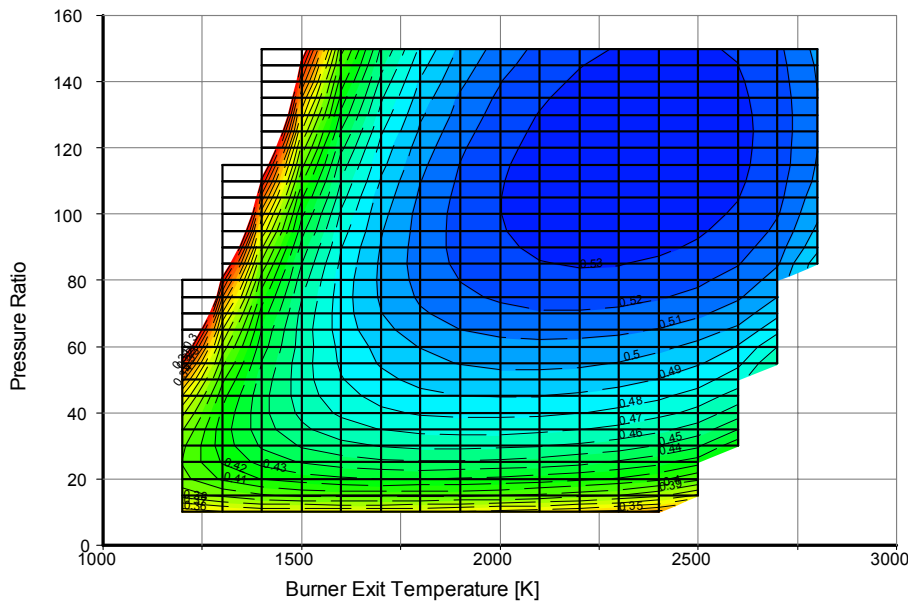


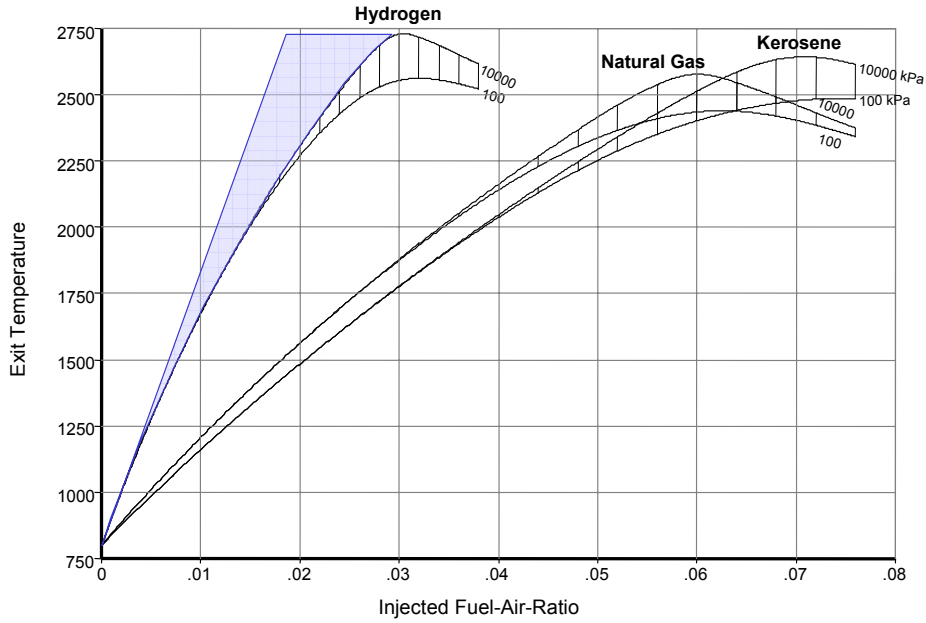
Fig. 5: Thermal Efficiency Defined with  $W_F * FHV$

The result of using this definition of thermal efficiency one gets what is reported in ref.1: There is an optimum of thermal efficiency at temperatures well below the stoichiometric limit! At a given pressure ratio of 40, for example, increasing burner exit temperature beyond 2000K would decrease thermal efficiency even if no cooling air is employed. This optimum is obviously caused by using  $W_F * FHV$  as denominator and therefore it is adequate to study the heat release process in the burner in some detail.

### 3.2 Temperature Increase in the Burner

In the schoolbook definition of thermal efficiency the fuel flow is implicitly assumed to be proportional to  $C_p * (T_4 - T_3)$ . In reality fuel flow respectively fuel-air-ratio is not proportional to  $T_4 - T_3$  as can be seen from fig. 6. For example, to get 2000K exit temperature with hydrogen as fuel would require a fuel-air-ratio of 0.0115 if the amount of fuel would be proportional to the temperature increase as the blue line indicates. However, in reality one needs the fuel-air-ratio of 0.015 – which is 30% more. Note that this effect has nothing to do with dissociation - up to 2000 K the exit temperature is independent from pressure, see fig. 6.

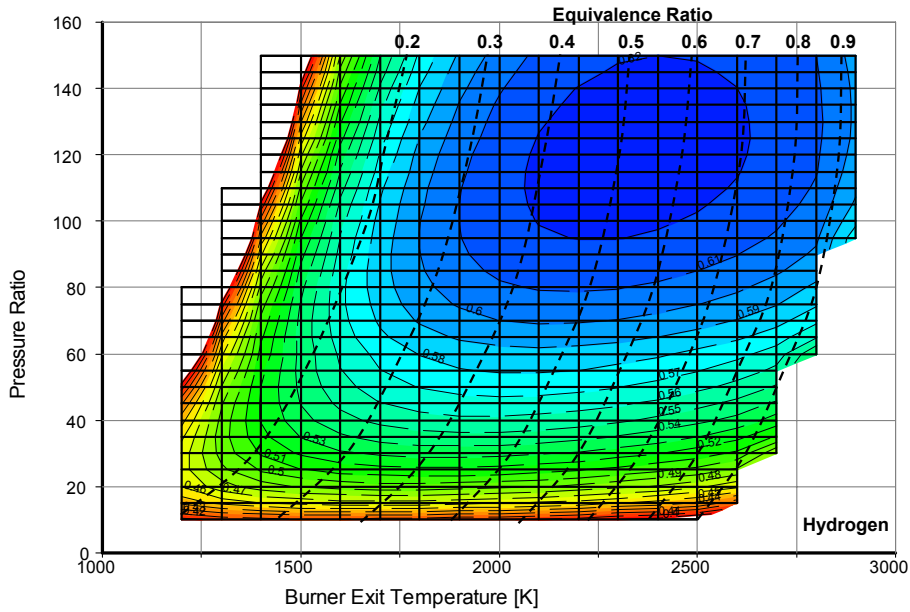
The fact that for achieving high temperatures one needs over-proportional amounts of fuel is the reason for the maximum thermal efficiency being at a temperature much lower than the stoichiometric value. This effect is independent from the fuel type as can be seen from a comparison of fig. 5 (fuel: Kerosene) with fig. 7 which was calculated with hydrogen as fuel.



**Fig. 6:** Burner Exit Temperature for Different Fuels ( $T_3=800K$ )

In figure 7 besides the lines for constant thermal efficiency also lines with constant equivalence ratio are shown. Equivalence ratio is defined as

$$ER = \frac{far}{far_{stoichiometric}}$$



**Fig. 7:** Thermal Efficiency with Hydrogen as Fuel with Lines of Constant Equivalence Ratio

The optimum thermal efficiencies as function of pressure ratio are found along the line  $ER = 0.5$ . The same correlation can be observed when lines for constant equivalence ratio are plotted into fig. 5. Thus the following generally valid statement can be made:

The maximum thermal efficiency of the simple gas turbine cycle with polytropic efficiencies equal to 0.9 and no cooling air simulation is found with fuel-air-ratios approximately equal to 50% of the stoichiometric value, independently from the type of hydrocarbon fuel burnt.

### 3.3 Effect of Component Efficiencies

All the cycle studies discussed above were performed with the same assumption about the quality of the turbo-machinery: polytropic efficiencies were always equal to 0.9 and no further losses were considered except that the amount of air leakage was set to be equal to the amount of fuel used.

What happens if the component efficiencies are different has been already reported in ref. 1: As the efficiencies are increased the point of maximum cycle efficiency shifts at constant pressure ratio to lower values of  $T_4$ . Fig. 8 shows this for the example of pressure ratio 60.

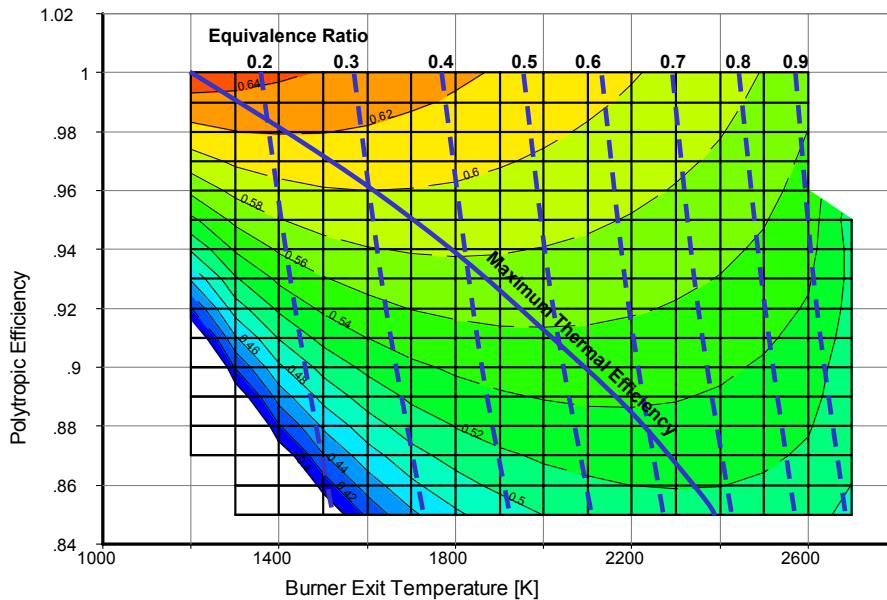


Fig. 8: Burner Temperature for Maximum Thermal Efficiency, Pressure Ratio = 60

### 3.4 Cooling Air Simulation

For more accurate simulations of course the amount of cooling air needed and the associated losses must be modeled adequately. Here we employ a rather simple method for estimating the amount of cooling air which correlates permissible mean metal temperature, cooling air amount, cooling effectiveness, cooling air temperature and  $T_4$ .

Only the cooling of the first turbine stage is considered. The vane cooling air is mixed with the main stream before the first rotor and thus the rotor entry temperature  $T_{41}$  is lower than the burner exit temperature  $T_4$ . With respect to rotor cooling the relative total temperature is the driving parameter. Without going into the details of an aerodynamic turbine design this temperature is approximated as  $0.9 \cdot T_{41}$ .

For finding an appropriate amount of cooling air the cooling effectiveness is used:

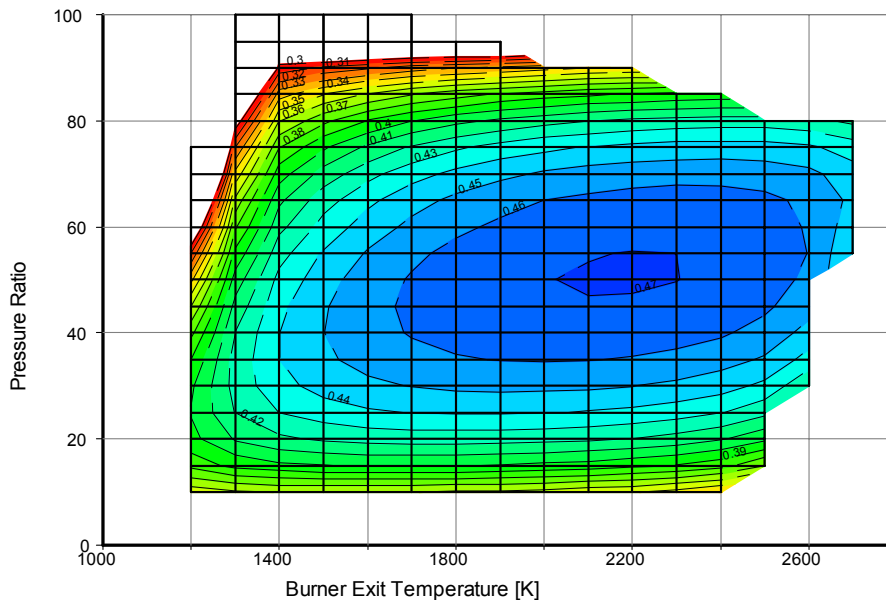
$$\eta_{cl} = \frac{T_{gas} - T_{metal}}{T_{gas} - T_{coolant}}$$

The amount of cooling air needed for achieving a certain cooling effectiveness depends on the design of the vane respectively blade cooling. For low cooling effectiveness it is sufficient to employ a design with convective cooling while for higher  $\eta_{cl}$  values film cooling is required. An approximate value for the amount of cooling air needed can be found from the empirical correlation which is taken from the GasTurb user's manual:

$$\frac{W_{cl}}{W_{gas}} = C * \frac{\eta_{cl}}{1 - \eta_{cl}}$$

This expression can also be found in ref. 2 (which contains also several more similar correlations) together with quite some physical background.

The constant C in the formula is set to 0.05 which yields a reasonable amount of cooling air over the full range in the parametric study. The result for thermal efficiency is shown in fig. 9.



**Fig. 9:** Thermal Efficiency with Cooling Air Simulation

Now we see the optimum at a place which is not far from a realistic cycle. The whole top right part – where high pressure ratios are combined with high temperatures – does no longer exist. The reason is the excessive amount of cooling air which is needed in this region; Fig. 10 shows the amount of NGV cooling air, the numbers for the rotor cooling air are somewhat smaller. The equivalence ratio at the optimum thermal efficiency is  $ER = 60\%$  and thus about 10% higher than that for the un-cooled cycle, see section 3.2.

If fig. 9 would show the design space of a real engine, and the only figure of merit would be the thermal efficiency then one would select from figure 9 as cycle design point the parameter combination  $P_3/P_2=50$  and 2200K. However, since the combination  $P_3/P_2=40$  with 1800K burner exit temperature yields only 1% less thermal efficiency than at the maximum it would in practice be the better choice because it requires significantly less design effort and cost.

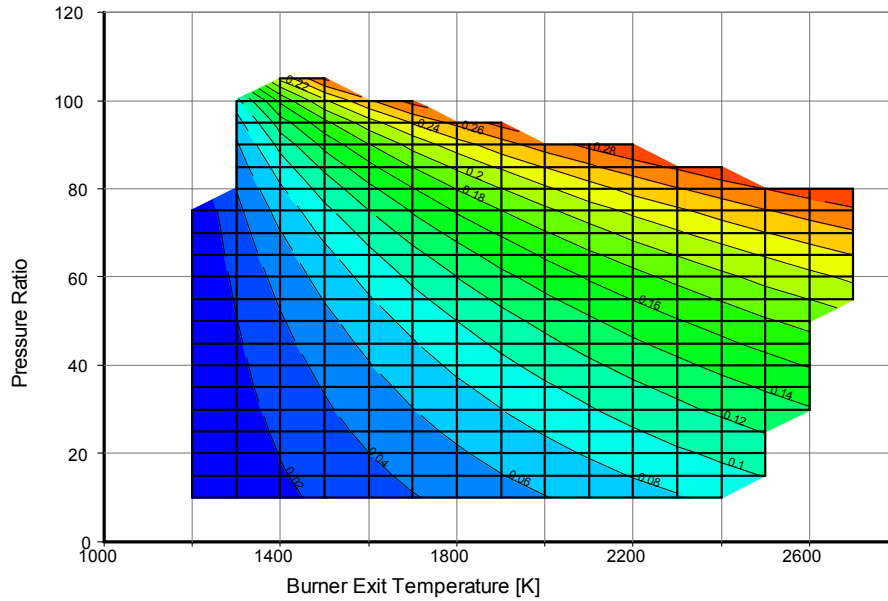


Fig. 10: NGV Cooling Air

### 3.5 Concluding Remarks

Optimizing an aircraft engine does not only ask for high thermal efficiency but also for low weight and low frontal area, in other words for high specific power per unit of mass flow. Specific power always increases with burner exit temperature and its maximum shows up at a significantly lower pressure ratio than that required for optimum thermal efficiency. This is illustrated in fig. 11 in which the dashed arrow indicates maximization of thermal efficiency while the solid arrow connects the maxima of specific power at any given burner exit temperature.

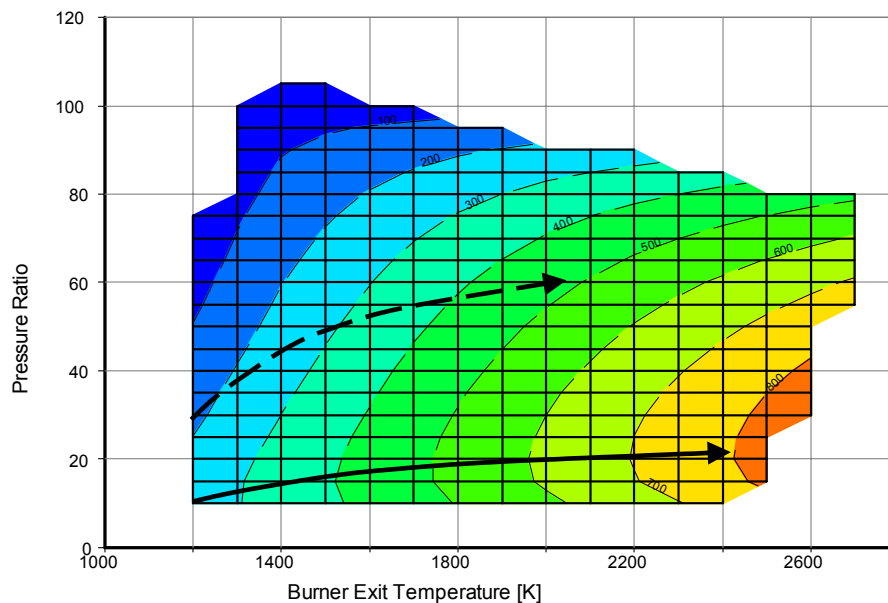


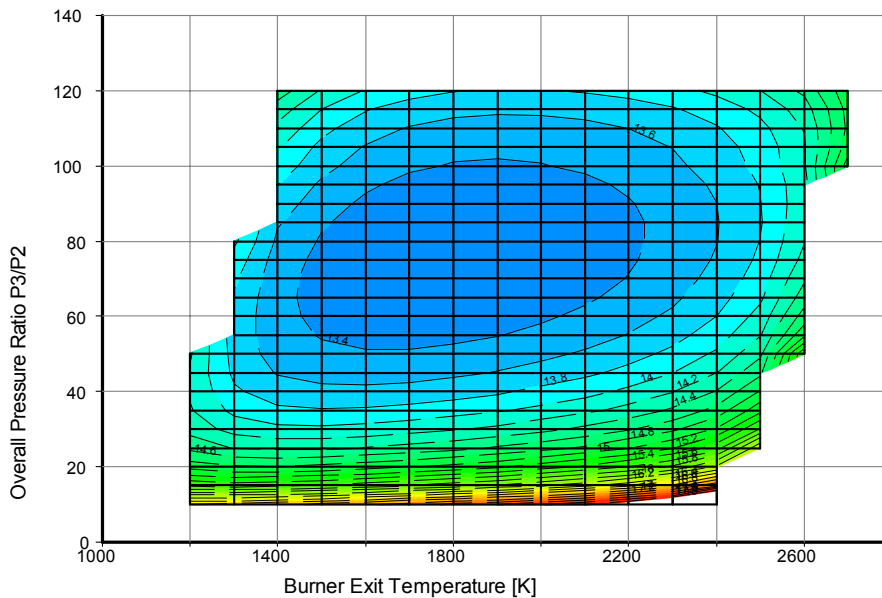
Fig. 11: Specific Power

The cycle studies discussed so far have all been done for sea level standard day conditions. As a consequence the trends and numbers shown are more directly applicable to power generation than to commercial aircraft propulsion. An aircraft engine will be optimized for

cruise conditions at 11km altitude, Mach 0.8, for example, where the engine inlet temperature is  $T_2=244\text{K}$  instead of  $288\text{K}$  at sea level.

Running an equivalent cycle study to that from fig. 9 for an unmixed flow turbofan yields the results plotted in fig. 12. The bypass ratio varies with the gas power produced by the core, the fan pressure ratio is kept constant and therefore the propulsive efficiency is (nearly) constant. This has the consequence that the shapes of the sfc-contour lines are the same as those for the thermal efficiency of the core.

The optimum burner temperature with respect to core thermal efficiency or specific fuel consumption is around  $1800\text{K}$ . The burner exit temperature of modern aero-engines is during cruise in the range of  $1500\dots1700\text{K}$ ; not much can be gained with respect to specific fuel consumption by further increasing the temperature at constant pressure ratio. Increasing core thermal efficiency is achieved most effectively by increasing pressure ratio relative to those of today's engines, not by increasing burner exit temperature towards the stoichiometric limit.



**Fig. 12:** Specific Fuel Consumption of a Turbofan at Cruise Conditions, Constant Propulsive Efficiency

Increasing propulsive efficiency by decreasing the fan pressure ratio goes with increasing bypass ratio and yields lower SFC at a given core thermal efficiency. However, the improvement potential for the specific fuel consumption is moderate since also the main element of the bypass process is already near to its practical limit for a conventional turbofan configuration.

Of course the improvement of component efficiencies and the reduction of the cooling air will remain also in future important goals of any engine maker. However, success in these directions is extremely difficult since at the same time economics require that the number of parts is reduced which has the consequence that the aerodynamic loading increases. There is not much room for improvement of the component efficiencies anyway since modern optimized 3D blade designs are already very good. Raising the efficiency of a low pressure turbine from say 93 to 94% would mean decreasing the losses by 15% and that certainly would be very ambitious. Consequently the trend in specific fuel consumption reduction of new aircraft engines over time which has been observed in the past will flatten out.

In summary it must be concluded that for commercial turbofan engines significant improvements of the specific fuel consumption of can only be expected if a new engine concept invalidates the findings presented in this paper.

### **Acknowledgement**

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## **4 REFERENCES**

- [1] Gas Properties as a Limit to Gas Turbine Performance  
R. C. Wilcock, J. B. Young and J. H. Horlock  
ASME GT-2002-30517, 2002
- [2] Limitations on Gas Turbine Performance imposed by Large Turbine Cooling Flows  
J. H. Horlock, D. T. Watson, T. V. Jones  
ASME 2000-GT-635, 2000
- [3] Exceeding 2000 K at Turbine Inlet: Relative Cooling with Liquid for Gas Turbines – Integrated Systems  
Constantin Sandu, Dan Brasoveanu  
ASME GT 2003-38031, 2003
- [4] Gas Turbine Performance Simulation with GasTurb™  
Joachim Kurzke  
[www.gasturb.de](http://www.gasturb.de)
- [5] Computer Program for Calculation of Complex Chemical Equilibrium Compositions and Applications.  
I. Analysis  
Sanford Gordon and Bonnie J. McBride  
NASA Reference Publication 1311, Oct 1994
- [6] Computer Program for Calculation of Complex Chemical Equilibrium Compositions and Applications.  
II. Users Manual and Program Description  
Bonnie J. McBride and Sanford Gordon  
NASA Reference Publication 1311, June 1996