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Description of Thermal Effects in Aero Engines by Matrices

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Abstract

A matrix method for simulation of material temperatures for given gas-temperature characteristics for use in performance programs is introduced. First of all, the matrix coefficients need to be determined from a reduced model derived from a previously performed detailed calculation or from measurements collected during engine tests. A procedure for identification of the matrix coefficients is presented. The matrix obtained can then be used to predict the system's behavior for other characteristics of gas temperature.

Changes in heat-transfer coefficients and / or material properties in the examined time envelope would normally result in changing matrix coefficients. The paper shows, that by restructuring the equations a form can be found, where the matrix remains constant as heat-transfer coefficients and material properties change. The method is applied to an HP-compressor rotor. The good agreement of reference solution and prediction given by the matrix method demonstrates the validity of the method.

Nomenclature

Symbols

T	[K]	temperature
a	[1/s]	matrix coefficient
b	[1/s]	matrix coefficient
c_p	[J/(kg K)]	heat capacity
k	[1/s]	constant
k*	-	constant
m	[kg]	mass
Δr	[m]	difference between two radii
A	[m^2]	area
Bi	-	Biot number
F	-	node in fluid
HP	-	high pressure
N	-	node in structure

Greek

α	[$W/(m^2 K)$]	heat-transfer coefficient
λ	[W/(m K)]	heat-conduction coefficient

Subscripts

f	number of nodes in fluid
matrix	solution from matrix model
n	number of nodes in structure
ref.	reference solution
ref. (m and l)	characteristic property of the system
CA	cooling-gas flow
G	gas flow
HG	main-gas flow
S	structure

Superscripts

-	average

1. Introduction

When the operating point of an aero engine is changed, this results first of all in a change of gas temperatures. Due to heat transfer between gas and structure the material temperatures are affected, too. Although the heat transfer is only a second-order effect in performance calculations there are some important consequences:

- Changing material temperatures cause thermal growth or contraction. As the material temperature of the disc, blade and casing do not change simultaneously, clearances in the machine will change. This has an impact on component efficiencies, compressor surge margin and, depending on the control system, also on temperature margin or net thrust of the engine. These thermal effects are of special importance for take-off, when after a fast acceleration from idle to take-off there is a high thrust demand. The take-off thrust is also influenced directly by the heat that is needed for heating up the structure and thus is not available for generating thrust.

- When the temperature change within the structure is non-uniform, inner stresses result. In discs e.g. the temperature increase after an acceleration from idle to take-off is generally faster at the outer radius than at the hub, because the heat transfer between main-gas flow and blade / inner shroud is more intensive than between cooling-gas flow and disc. Therefore, high stresses within the disc result, when after acceleration high centrifugal stress caused by high speed adds up with thermal stresses associated with the temperature difference between rim and hub. This needs to be taken into account when the disc geometry is designed. As a secondary effect, these stresses cause additional clearance changes.

From all these effects the necessity arises to describe and predict the heat transfer within transient performance calculations. One possibility to do that is the application of a state-space approach to describe the thermal effects by matrices as depicted e.g. in [1] and [4]. But when using this strictly mathematical approach some problems arise concerning the universal validity of the matrices and the handling of changing material properties and heat transfer coefficients. This paper describes how these problems can be solved through adaptation of the basic equations to the physical problem. The data needed to build the state-space model is either generated by detailed thermal finite-element calculations or consists of test results. In the first case, only a few temperature nodes of the finite-element calculation are later used to create a reduced model for the description of the thermal effects in a performance program. The goal of the model reduction is to generate a reduced model that still sufficiently represents the behavior of the original model but which is of lower order and only uses task-specific quantities [5]. The parameters of this reduced model are generated by means of identification. In the second case, data provided by test results is used to identify the parameters of the model. The focus of this paper is on heat transfer and the resulting change of material temperatures. A detailed discussion of the simulation of thermal growth and stresses using the results generated in this paper is given in [3].

2. Basic ideas of the method

The method assumes, that in a system described by n structure temperatures and f gas temperatures the change of a structure temperature over time is a linear function of all these temperatures:

$$\begin{bmatrix} \dot{T}_{S1} \\ \dot{T}_{S2} \\ \cdot \\ \dot{T}_{Sn} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdot & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdot & a_{2,n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n,1} & a_{n,2} & \cdot & a_{n,n} \end{bmatrix} \cdot \begin{bmatrix} T_{S1} \\ T_{S2} \\ \cdot \\ T_{Sn} \end{bmatrix} + \begin{bmatrix} b_{1,1} & b_{1,2} & \cdot & b_{1,f} \\ b_{2,1} & b_{2,2} & \cdot & b_{2,f} \\ \cdot & \cdot & \cdot & \cdot \\ b_{n,1} & b_{n,2} & \cdot & b_{n,f} \end{bmatrix} \cdot \begin{bmatrix} T_{G1} \\ T_{G2} \\ \cdot \\ T_{Gf} \end{bmatrix} \qquad (1)$$

If all coefficients of the matrices and all gas and structure temperatures are given, this equation allows calculation of the present gradients of the structure temperatures. By integration, the structure temperatures can be calculated as functions of time for prescribed gas temperatures. This is how the method is intended to be used: The coefficients of the matrices are known, gas-temperature characteristics are given (e.g. as results from the actual performance calculation) and structure element temperatures are to be determined.

At first the coefficients of the matrices are not known. If characteristics of gas temperatures and corresponding structure temperatures are known – e.g. from a measurement or a calculation done by the specialist department - the task is to optimize the matrix coefficients, in order to approximate the given characteristics of the structure temperatures as good as possible. These coefficients can then be used to predict the system’s behavior for other characteristics of gas temperature. The next paragraph demonstrates, that some extra efforts are necessary to ensure realistic predictions with this method.

3. Application of the method to a simple test case

The test case used in this paragraph is an axially symmetric structure with the typical shape of a disc, see Fig. 1. Heat transfer only occurs at the cylindrical surfaces at the outer rim and at the hub. At the outer radius there is contact with the main-gas flow (temperature T_{HG}), at the inner radius there is contact with the cooling-gas flow (temperature T_{CA}).

The material properties of the structure as well as the heat-transfer coefficients were assumed to be constant for this example. In paragraph 3.4 the effects of varying properties on the matrix coefficients will be discussed.

In order to create a reference solution, the whole structure was split up into twenty elements as shown in Fig. 1. Applying the fundamental equations for heat transfer and heat conduction, the temperature for each of these elements was calculated as a function of time.

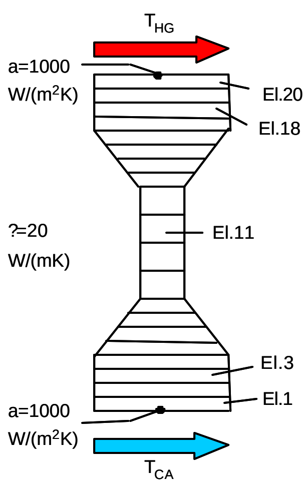

Fig.1: Test case geometry.

The example examines the structure temperatures within a period of 1000 seconds. The following characteristics of T_{HG} and T_{CA} are prescribed (see Fig. 2).

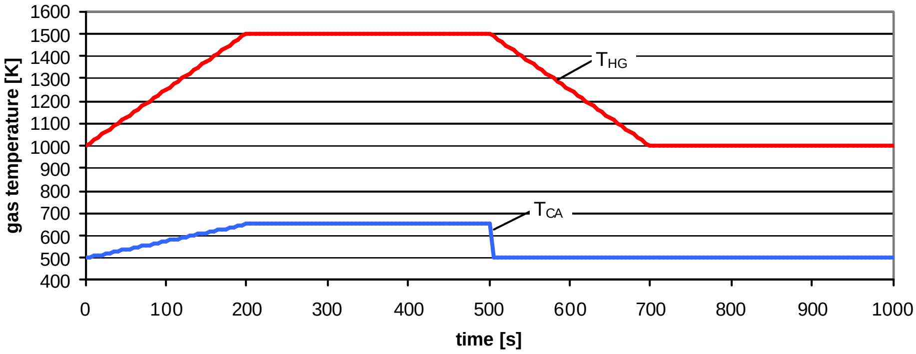

Fig.2: Test case gas -temperature characteristics

3.1 Determination of the matrix coefficients for best curve fit

Taking the model of the previous paragraph as a basis a reduced model was created so as to simulate the use of the method for finite-element calculations. This reduced model can also be seen as the model used with data from test results. For the matrix method only the temperatures of the elements 3, 11 and 18 from the previous calculation are taken into account. An attempt has been made to represent the temperature gradient of these elements as a function of the three element temperatures and the two gas temperatures:

$$\begin{bmatrix} \dot{T}_3 \\ \dot{T}_{11} \\ \dot{T}_{18} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} T_3 \\ T_{11} \\ T_{18} \end{bmatrix} \qquad (2)$$

$$+ \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} \cdot \begin{bmatrix} T_{HG} \\ T_{CA} \end{bmatrix}$$

The matrix coefficients are determined row by row with a standard regression technique (e.g. shown by Ljung [2]). From the reference characteristic $T_3(t)$ the gradient $\dot{T}_{3,ref.}(t)$ can be derived for discrete points in time between 0s and 1000s. Between this “true” gradient at a certain point in time and the gradient calculated by the matrix approach given above a difference remains which can be expressed as

$$\Delta\dot{T}_3 = \dot{T}_{3,ref.} - a_{11}\cdot T_3 - a_{12}\cdot T_{11} - a_{13}\cdot T_{18}$$

$$- b_{11}\cdot T_{HG} - b_{12}\cdot T_{CA}\,. \qquad (3)$$

The magnitude of this difference is dependent on the choice of the coefficients. These differences are squared and added up for all the points in time taken into account. This sum of residual squares is to be minimized by appropriate choice of the coefficients. So the partial derivatives of this sum with respect to each of the coefficients need to be zero. This yields a set of five linear equations for the five coefficients which can be solved. In the same manner, the coefficients in rows two and three of the matrices are determined by the method of least residual squares.

3.2 Check of the universal validity of the matrix

To check the universal validity of the matrices, the coefficients a_{11} to a_{33} and b_{11} to b_{32} have been calculated only for the time interval [0s, 500s] according to the method described in chapter 3.1. The resulting matrices have been used to calculate the material temperatures for the same maneuver in the whole time interval [0s, 1000s]. Fig. 3 compares temperatures T_3, T_{11} and T_{18} of the reference solution with the one calculated from the matrix method. The continuous lines represent the reference solution obtained from the twenty-element system. The symbols represent the solution calculated by the matrix method. The conclusions from this comparison are:

- There is a good agreement in the range up to 500 s, which has been used for the identification of the matrix elements.
- There are significant deviations in the time interval [500s, 1000s]. Obviously, the matrix identified on a case with rising gas temperatures cannot be applied to a case with decreasing gas temperatures.
- As will be discussed in detail in paragraph 3.4, matrices with constant coefficients are not appropriate for exact prediction of structure temperatures whenever material properties (i.e. heat-conduction coefficient or heat capacity) vary over the period observed. The same is true, if heat-transfer coefficients change. But in the present case all of these properties have been constant and still the results in the second time interval are not satisfactory.

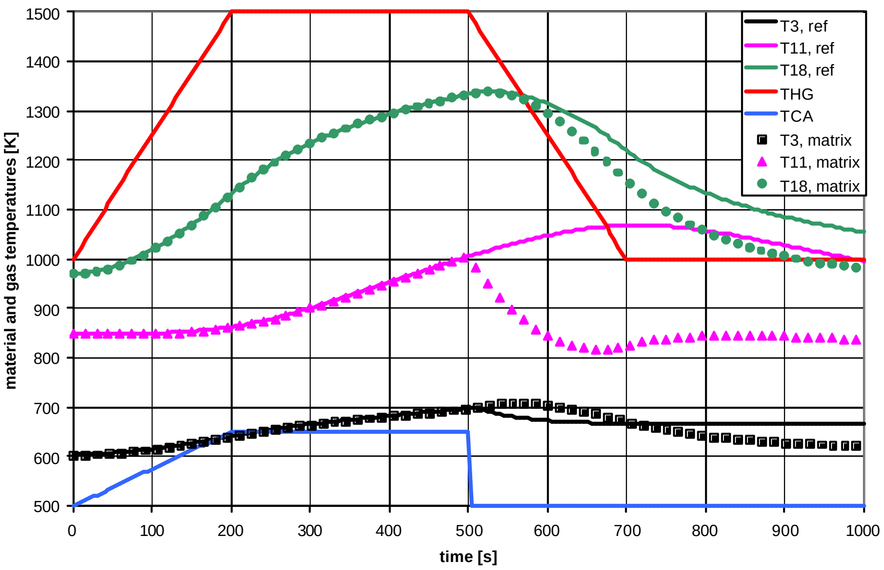

Fig.3: Comparison of reference and simulated material temperatures (simple method)

3.3 Improving the universal validity of the matrix

According to equation (2) the temperature gradient $\dot{T}_3$ is a function of all three structure temperatures and both gas temperatures. This is not in accordance with the equations used in a system of replacement structures. In the twenty-element system used to create the reference solution (see Fig. 1) the temperature change of element one, for example, can only be influenced by two heat flows: heat transfer with cooling air and heat conduction with its neighbor element two. The heat transfer is proportional to the temperature difference T_{CA}-T_1 and the heat conduction to the temperature difference T_2-T_1. So all other element temperatures from T_3 to T_{20} as well as the hot gas temperature will have no direct influence on $\dot{T}_1$.

An equivalent approach for the reduced model, where only the temperatures T_3, T_{11} and T_{18} are taken into account is:

$$\left.\begin{aligned}\dot{T}_3 &= k_1 \cdot (T_{11} - T_3) + k_2 \cdot (T_{CA} - T_3)\\ \dot{T}_{11} &= k_3 \cdot (T_3 - T_{11}) + k_4 \cdot (T_{18} - T_{11})\\ \dot{T}_{18} &= k_5 \cdot (T_{11} - T_{18}) + k_6 \cdot (T_{HG} - T_{18})\end{aligned}\right\} \quad (4)$$

This system of equations replaces the former approach given by equation (2). The number of coefficients to be determined by the regression analysis is reduced from 15 in equation (2) to 6 in equation (4). Equation (4) includes some physics that are missing in equation (2): On the one hand, heat flows and temperature change of elements are dependent on driving temperature differences, not on single temperatures. On the other hand, heat exchange is not allowed between arbitrary elements and gas flows but only where contact occurs.

Rewriting eq. (4) with the matrix approach used in eq. (2) gives a better understanding of the advantages introduced.

$$\begin{bmatrix}\dot{T}_3\\ \dot{T}_{11}\\ \dot{T}_{18}\end{bmatrix} = \begin{bmatrix}-(k_2+k_1) & k_1 & 0\\ k_3 & -(k_3+k_4) & k_4\\ 0 & k_5 & -(k_5+k_6)\end{bmatrix} \cdot \begin{bmatrix}T_3\\ T_{11}\\ T_{18}\end{bmatrix} + \begin{bmatrix}0 & k_2\\ 0 & 0\\ k_6 & 0\end{bmatrix} \cdot \begin{bmatrix}T_{HG}\\ T_{CA}\end{bmatrix} \quad (4,a)$$

Compared with eq. (2), it is obvious that now the matrix elements of each row are coupled or explicitly set to zero, as stated above.

Another regression analysis is performed to determine the coefficients k_1 to k_6 in equation (4) that give optimum approximation of the material temperatures in the time interval [0s, 500s]. Again, these coefficients that have only been calibrated on a period with rising gas temperatures, are then used to predict the temperature of the three elements in the complete time window up to 1000s. The results are given in Fig. 4.

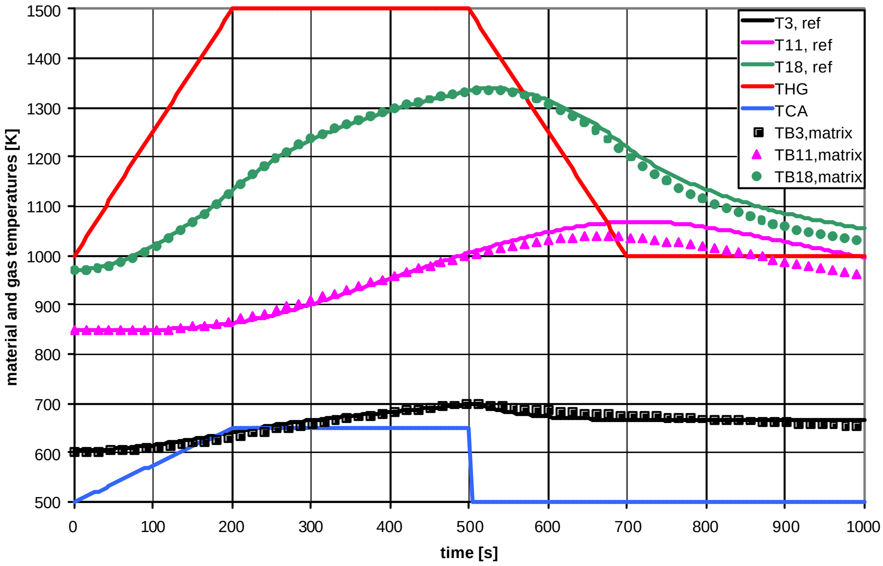

Fig. 4: Comparison of reference and simulated material temperatures (improved method)

As shown in Fig. 4 there are still deviations in the part that has not been used for identification of the matrix coefficients, but they are much smaller than in the previous case. If the whole time interval from 0s to 1000s is used for calculating the coefficients, there is a good agreement in the complete period. The coefficients are then suitable both for rising and decreasing gas temperatures and thus, the system behavior for arbitrary characteristics of the gas temperatures can be described with good accuracy.

3.4 Rewriting the equations to account for varying material properties and heat-transfer coefficients

The coefficients k_1 to k_6 in equation (4) are dependent on heat-conduction coefficient ? and heat capacity c_p of the structure, on the two heat-transfer coefficients a_{CA} and a_{HG} and on the geometry. If the behavior of an engine component in different operating states is to be described, the geometry is fixed. In this case, the dependence of the matrix on the other four parameters remains. The material properties ? and c_p are functions of the material temperature and can vary by 30% between different operating points. The heat-transfer coefficients are dependent on the state of flow in the two gas streams and can vary even more than the material properties. As a consequence, the coefficients k_1 to k_6 in our example would need to be calculated and stored for different combinations of the four varying parameters in order to ensure good prediction of material temperatures for arbitrary transient operations. Assuming 5 values for each of the parameters need to be investigated to describe the full engine operating range in a sufficiently accurate manner, this means that all coefficients need to be calculated and stored $5^4 = 625$ times. In this paragraph it is shown, how by rewriting the equations the dependency of the coefficients on material properties and heat-transfer coefficients can be eliminated.

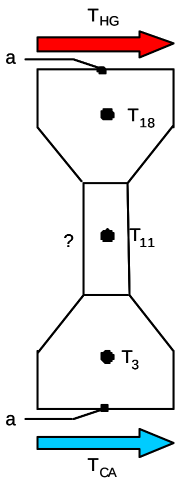

Fig.5: 3 Element system

As a starting point, the equation is given that describes the gradient of T_3, if the structure in our example is substituted by only three elements with the temperatures T_3, T_{11} and T_{18}, see Fig. 5.

$$\left.\begin{aligned} \dot{T}_3 = & \frac{l \cdot A_{3,11} / \Delta r_{3,11}}{m_3 \cdot c_p} \cdot (T_{11} - T_3) \\ & + \frac{a_{CA} \cdot A_{CA}}{m_3 \cdot c_p} \cdot (T_{CA} - T_3) \end{aligned}\right\} \quad (5)$$

This is equivalent to the first line of equation (4). Instead of the constants k_1 and k_2 in equation (4) we find in equation (5) all the physical terms influencing the value of the constants k_1 and k_2. The statement made at the beginning of the paragraph, that the constants k_1 to k_6 in equation (4) are dependent on geometry, material properties and heat-transfer coefficients, is confirmed by equation (5).

Equation (5) is now transformed to

$$\left.\begin{aligned}\frac{m_{ref}\cdot\bar{c}_p}{\bar{l}\cdot l_{ref}}\cdot\dot{T}_3 &= \frac{m_{ref}}{m_3}\frac{l}{\bar{l}}\frac{\bar{c}_p}{c_p}\frac{A_{3,11}}{\Delta r_{3,11}\cdot l_{ref}}\cdot(T_{11}-T_3)\\ &+\frac{m_{ref}}{m_3}\frac{\bar{c}_p}{c_p}\frac{a_{CA}\cdot A_{CA}}{\bar{l}\cdot l_{ref}}\cdot(T_{CA}-T_3)\end{aligned}\right\} \quad (5a)$$

The values with index "ref" are characteristic properties of the system. m_{ref} might e.g. be the mass of the complete structure, l_{ref} the tip radius. $\bar{l}$ and $\bar{c}_p$ are material properties calculated for the mean temperature of the whole structure. By this transformation, the coefficients on the right hand side of the equation have become dimensionless. Besides, the first coefficient can be assumed to be constant: Some of the ratios just contain geometric data. For constant geometry they will not change. The term $l/\bar{l}$ is the ratio of local / average heat-conduction coefficient that depends on the ratio of local / average temperature. Obviously, this ratio is not necessarily constant, but the general trend will be, that if the local temperature rises the average temperature rises too, so that the assumption that $l/\bar{l}$ is constant might be suitable for practical purposes. The same is true for the ratio $\bar{c}_p/c_p$. Normally both properties ? and c_p increase with rising temperature. Therefore, potential changes in the ratio $l/\bar{l}$ will at least partially be compensated by changes in the ratio $\bar{c}_p/c_p$.

The second coefficient includes the term a_{CA}?A_{CA}/(? ?l_{ref}) which is a Biot number. The Biot number is a dimensionless number characterizing the ratio between heat transfer and heat conduction. The term in the equation above will be written as Bi_{CA} in the following. In the equation for the change of T_{18} an equivalent term occurs for the hot-gas flow Biot number which will be written as Bi_{HG}.

Using these definitions the new set of equations, replacing equation (4) can be written as:

$$\begin{aligned}\frac{m_{ref}\cdot\bar{c}_p}{\bar{l}\cdot l_{ref}}\cdot\dot{T}_3 &= k_1^*\cdot(T_{11}-T_3)+k_2^*\cdot Bi_{CA}\cdot(T_{CA}-T_3)\\ \frac{m_{ref}\cdot\bar{c}_p}{\bar{l}\cdot l_{ref}}\cdot\dot{T}_{11} &= k_3^*\cdot(T_3-T_{11})+k_4^*\cdot(T_{18}-T_{11}) \quad (6)\\ \frac{m_{ref}\cdot\bar{c}_p}{\bar{l}\cdot l_{ref}}\cdot\dot{T}_{18} &= k_5^*\cdot(T_{11}-T_{18})+k_6^*\cdot Bi_{HG}\cdot(T_{HG}-T_{18})\end{aligned}$$

In this form only two terms occur on the right hand side of the equations, that will vary during transient operation of the engine. These are the two Biot numbers. The coefficients k1* to k6* are constant. If the regression analysis is performed accordingly, only one set of coefficients k1* to k6* will be necessary for describing the transient system behavior. This will be demonstrated in the next paragraph.

4. Application of the method to a HP-compressor rotor

The method now is applied to the rotor of an HP-compressor. The thermal behaviour of this rotor has been predicted with the tools used in the engine design process at MTU. In Fig. 6 the rotor geometry and the location of the nodes are shown. Nodal points within the fluid are marked by the letter "F", nodal points within the structure by "N". The temperature characteristics for all these nodes have been supplied by the specialist department for an acceleration of the engine from idle to take-off conditions followed by a deceleration to idle conditions at sea level. The calculation performed in the specialist department covers about 300 nodes in a finite-element calculation: A distribution of the fluid temperature within the hot-gas flow path is modeled, a temperature distribution within the space between adjacent discs is calculated, heat-conduction effects between adjacent rotors via the drum are taken into account.

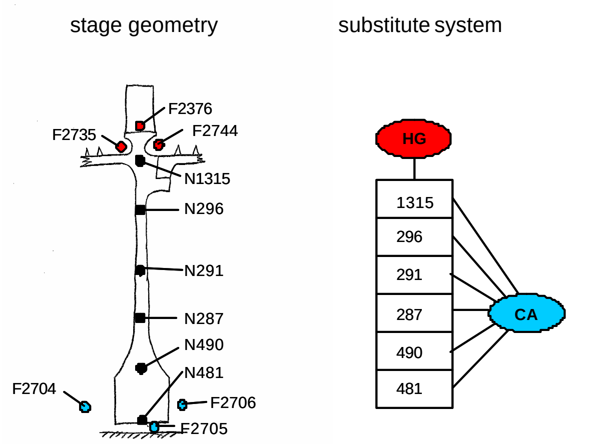

Fig. 6: HP-rotor geometry with node positions and replacement structure

On the right hand side of Fig. 6 the substitute system which is the basis for determination of the matrix is shown. The structure is represented by six radially-arranged elements. Element 481 only has heat conduction with element 490, element 1315 only with element 296. All other structure elements experience heat conduction with two neighbor elements. There is heat transfer between all structural elements and the cooling air. The cooling-air temperature is represented by one value only, which is calculated as the average of F2704 to F2706. The same is true for the hot-gas temperature, which is calculated as mean value of temperatures at F2735, F2376 and F2744.

The dimensionless matrix coefficients according to paragraph 3.4. were determined by applying the method of least squares on the whole time interval from 0s to 2000s. The behavior of the material temperatures was then simulated with these matrix elements for prescribed gas temperatures.

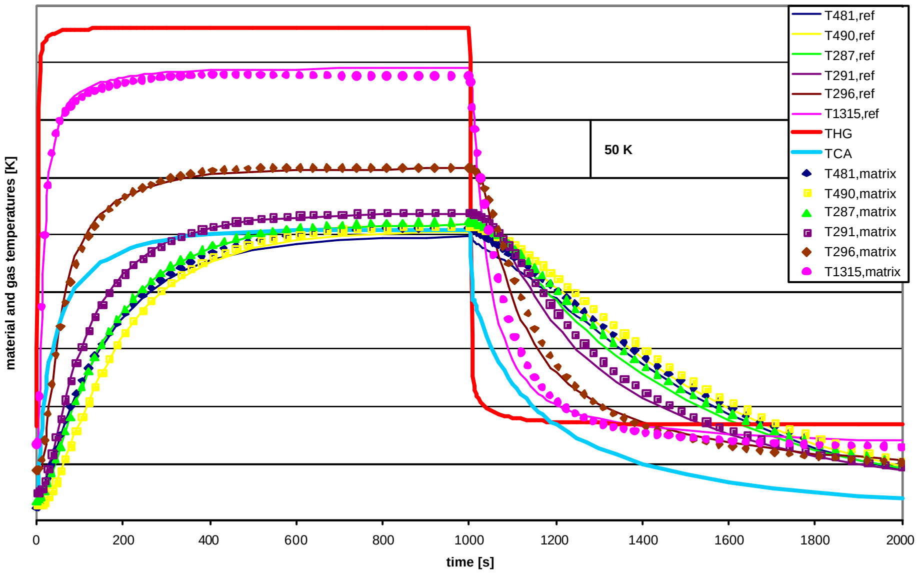

Fig. 7: Comparison of reference and simulated material temperatures (HP-rotor) for acceleration and deceleration

The original temperature characteristics supplied by the specialist department (reference) and the results of the simulation with the matrix method are compared in Fig. 7. There is a good agreement of reference and simulation in the whole time range. The small deviations can be explained by the simplifications made.

It is important to note that the heat-conduction coefficient varies by approximately 30%, heat capacity by about 15% during this maneuver. The heat-transfer coefficients increase from idle to take-off conditions by a factor between four and five. Without the new approach derived in paragraph 3.4. it would have been necessary to identify several sets of matrix coefficients, each valid in a small time window, where ?, cp and a do not vary too much. The new method allows us to identify only one set of matrix coefficients based on the system's behavior in the complete time interval [0s, 2000s].

In order to check the universal validity of the method, the matrix identified with the maneuver shown in Fig. 7 was used to simulate a test-case mission starting at idle followed by take-off, cruise, flight idle, reverse thrust and taxi. The results generated with the matrix method are shown in Fig. 8. Although the matrices have been determined with a different maneuver, the results of the test-case mission calculated with those matrices (dotted lines in Fig. 8) show a good agreement with the reference solution. Especially those mission parts, which have not been included in the identification process (cruise and reverse) are close to the reference solution. This demonstrates, that the identified matrices are not only valid for the mission used during the determination of the coefficients but for nearly arbitrary maneuvers.

4.1 Some comments on the modeling choice

The model used in paragraph 4 works with one cooling-air temperature only, although the temperatures at three nodes have been provided by the specialist department. Alternatively, it would have been possible to simulate contact of the structural elements with several of these cooling air nodes. In this case, additional matrix coefficients had to be identified.

The differences between the temperatures at the nodes F2704, F2705 and F2706 are relatively small. If two or all of them were identical in the complete time envelope, the set of equations for determination of the matrix coefficients would no longer be linearly independent and could not be solved. If the temperatures are not identical but close to each other, the equations can be solved, but the results are not reliable (only the sum of the identified cooling-air coefficients is). To avoid these problems an average cooling-air temperature has been used. The same is valid for the hot gas temperature.

4.2 Trouble shooting

If the formulae are arranged like in equation (6) all coefficients must have positive values. If the matrix identification method entails negative values for some coefficients, the quality of the approximation of the reference characteristics with the matrix coefficients should be checked. If it is satisfactory, there might be a situation as described above for the cooling air: There are several heat exchange partners for an element with similar temperature level. It is hard for the identification algorithm to identify the individual share of each partner, but the sum effect is described sufficiently.

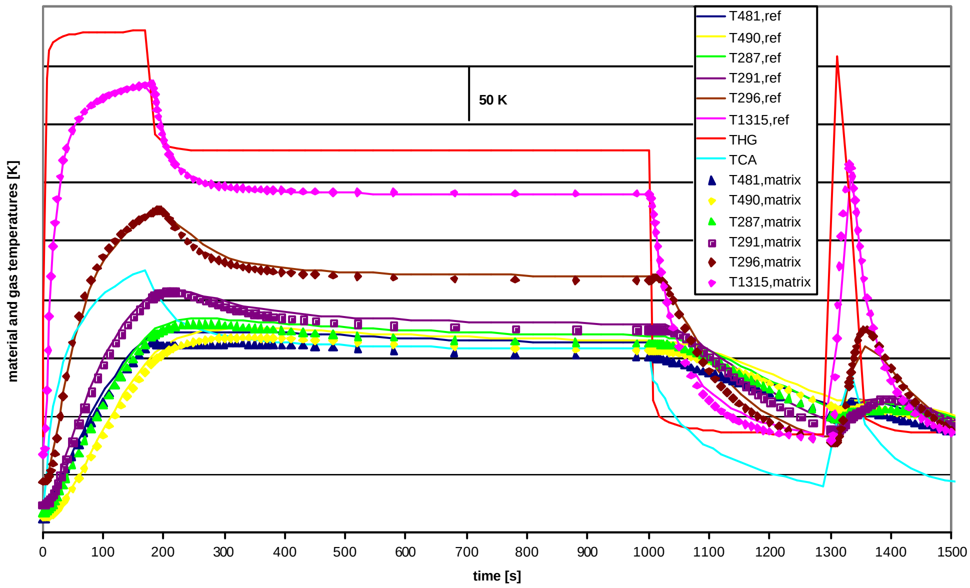

Fig. 8: Comparison of reference and simulated material temperatures (HP-rotor) for the test-case mission

This is why the approximation of the system's behavior works. Using such a matrix to predict the thermal behavior for a different case may give wrong results, when temperature levels that have been similar for the identification case no longer are.

If the approximation results are not satisfactory, the following points can be checked: Does the model take all significant possibilities of heat exchange into account? Is the number of elements used sufficient to model the system behavior in the required quality? Is the set of reference data consistent?

5. Conclusion

A matrix method for simulation of material temperatures for given gas-temperature characteristics for use in performance programs has been introduced. As a first step the matrix coefficients were determined from a reduced model that can either be derived from a previously performed detailed finite-element calculation or from measurements collected during engine tests. The procedure to identify the matrix coefficients was presented. Changes in heat transfer coefficients and / or material properties in the examined time envelope would normally result in changing matrix coefficients. It was shown, that by restructuring the equations a form can be found, where the matrix remains constant as heat-transfer coefficients and material properties change.

The matrix obtained can now be used to predict the system's behavior for other characteristics of gas temperature. The method was successfully applied to an HP-compressor rotor. The results generated in this paper provide the basis for a detailed discussion of the simulation of thermal growth and stresses for use in performance programs given in [3].

6. Acknowledgement

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7. Literature

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