Cyclic Mixed-Mode Crack Propagation due to Time-dependent Multiaxial Loading in Jet Engines

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Abstract. Components in jet engines are subject to time-dependent multiaxial loading. This creates a time-varying mixed-mode stress state at the crack tip. Mixed-mode loading leads to out-of-plane crack propagation and has been treated in previous articles [1,2]. This paper concentrates on coping with the time-dependent character. Key issues are the crack propagation rate and the crack propagation direction in three-dimensional space. In order to determine the prevalent crack propagation direction the dominant loading case is determined based on its crack propagation rate. Then, a mixed-mode equivalent K-factor is calculated for all other loading cases based on the closeness of their associated crack propagation direction with the dominant one. Subsequently, a cycle extraction is performed on the crack propagation rate for all loading cases. The extracted cycles are processed based on their mimimum and maximum equivalent K-factor and the maximum temperature. The mission crack propagation rate consists of the sum of the rate of all extracted cycles.

Introduction

Crack growth in real engine components usually exhibits mixed-mode, i.e. the crack does not necessarily stay in-plane. Since most crack propagation tests are performed under mode-I conditions, a procedure has to be defined to reduce the mode-I, mode-II and mode-III stress intensity (K) factors to just one equivalent K-factor which can be used in the well-known Paris crack propagation law (or any equivalent law). Furthermore, a bending and twist angle has to be calculated in order to determine the local crack propagation angle. Several criteria fulfilling the above conditions have been formulated in the past [1,2]. They often lead to similar results, however, due to the limited amount of mixed-mode testing further verification is needed. In the present paper it is assumed that the mixed-mode crack propagation procedure of [1] is used for the evaluation of the crack propagation rate and direction of any single cycle. The primary objective of the paper is to extend this procedure to the evaluation of complete missions. A mission is characterized by a sequence in time of loading points, each leading to a different mixed-mode state at any point along the crack front. An example of such a mission is a complete flight of an airplane, leading to time-dependent temperature and stress conditions at any location in its jet engines. Focusing on one location, the resulting mixed-mode state changes in time leading to several issues to be solved: what is the parameter to be used in the cycle extraction procedure and how does one cope with the different crack propagation angle of the lower and upper state of the extracted cycle. The next sections try to answer these questions.

Equivalent K-factor and crack propagation angles

The MTU mixed-mode crack propagation criterion[1] is based on a local analysis of the linear elastic asymptotic stress field along the crack front. Denoting the cylindrical coordinates in a local coordinate system at a location along the front by r, θ and z, the asymptotic stress field $\sigma_{ij}(r, \theta)$ depends on r only through a factor of $r^{-1/2}$. Therefore, multiplying this stress field by $r^{1/2}$ leads to the self-similar stress field $\sigma_{ij}^{*}(\theta)$. Notice that this stress field has the dimension of stress intensity factor. The eigenvalues corresponding to the self-similar stress tensor are denoted by $\sigma_1^{*}(\theta) \geq \sigma_2^{*}(\theta) \geq$

$\sigma_3^*(\theta)$, and the principal planes are described by $\varphi_1(\theta)$, $\varphi_2(\theta)$, $\varphi_3(\theta)$ (bending angles) and $\psi_1(\theta)$, $\psi_2(\theta)$, $\psi_3(\theta)$ (twist angles, Fig. 1). It is assumed that crack propagation will occur in a principal plane through the crack tip perpendicular to the largest principal self-similar stress. The condition that the principal plane must contain the crack tip is equivalent to postulating that the bending angle equals θ. So we are looking for $j \in \{1,2,3\}$ and $\theta^0 \in [-\pi/2, \pi/2]$ such that

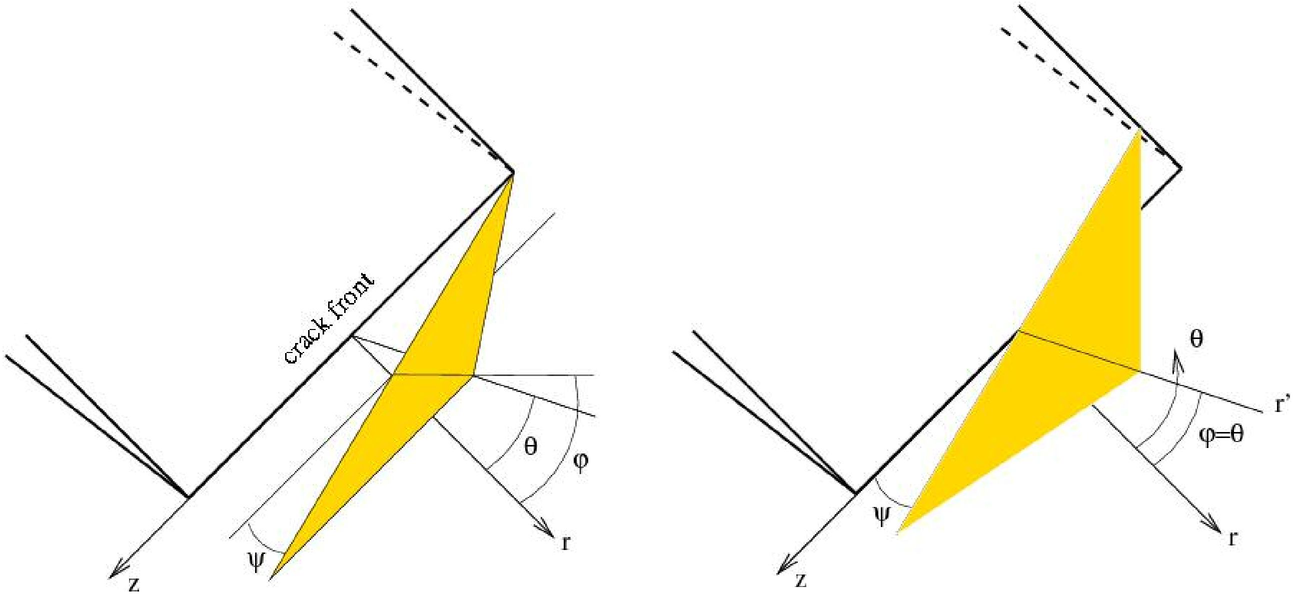

Figure 1: Principal plane not containing (left)/containing (right) the crack tip

$$\sigma_j^*(\theta^0) = \max\{\sigma_i^*(\theta) \mid \theta = \varphi_i(\theta)\},\ i \in \{1,2,3\}. \tag{1}$$

θ^0 is the bending angle of the crack propagation at the location considered. Generally speaking, the crack propagation is characterized by the largest principal stress, which is defined as the equivalent K-factor $K_{eq} \equiv \sigma_j^*(\theta^0)$, the normal vector on the crack propagation plane $\vec{n} \equiv \vec{n}_j(\theta^0)$, the bending angle $\varphi \equiv \varphi_j(\theta^0) = \theta^0$ and the twist angle $\psi \equiv \psi_j(\theta^0)$. By rotating the original crack plane φ radians about the local z-axis and subsequently ψ radians about the rotated r-axis r', one obtains the final position of the crack propagation plane. The complete set of principal stresses (not just the equivalent K-factor) and the complete set of principal directions is denoted by $\sigma_i^0 \equiv \sigma_i^*(\theta^0)$ and $\vec{n}_i^0 \equiv \vec{n}_i(\theta^0)$, respectively. These quantities depend on z (the location along the crack front) and time. So, up to now we have defined for each location along the crack front and for each time data point an equivalent K-factor and a direction the crack would like to take. However, in reality the time data points are not independent since cycle extraction links them as pairs, and the crack propagation direction for the upper and lower data point in a cycle should be the same. Therefore we are looking for a dominant crack propagation direction.

Identifying a dominant crack propagation direction

For the sake of the identification of a dominant crack propagation direction the standard Paris cyclic crack propagation law is taken in the form

$$\frac{da}{dN} = 10^{-7}\left(\frac{\Delta K_{eq}}{\Delta K_z}\right)^n \text{ (da/dN in m/cycle, K-factors in MPa.m}^{1/2}\text{),} \tag{2}$$

where ΔK_z and n are temperature-dependent material constants. Since the equivalent K-factor and the temperature are known for all times, one can plot the crack propagation rate for a given crack front location assuming 0-max cycles, i.e. $\Delta K_{eq} = K_{eq}$ (Fig. 2, dropping the constant factor 10^{-7}).

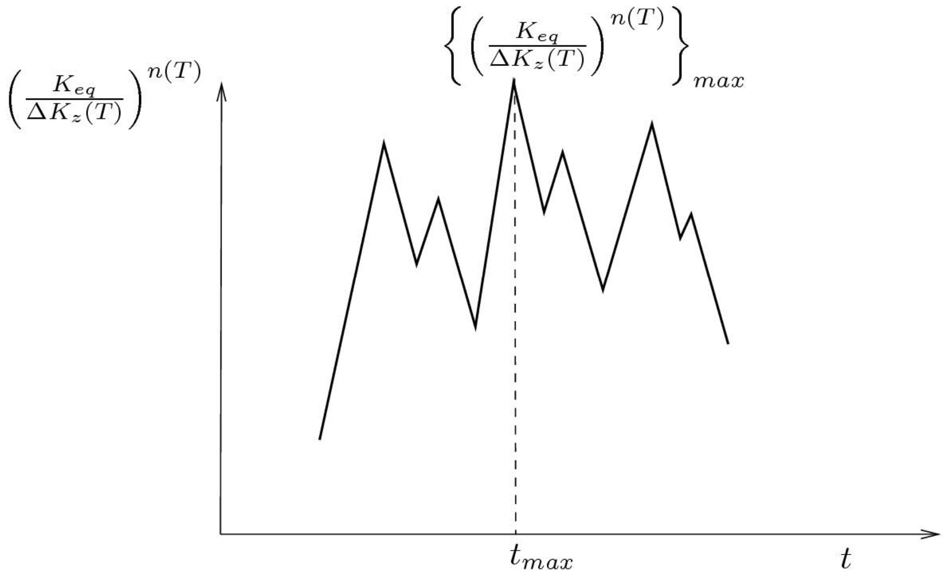

Figure 2: Definition of the dominant crack propagation time point

The time at which the crack propagation rate is maximum is called $t_{\max}$. The crack propagation direction $\theta^0(t_{\max})$ is considered as the dominant direction. The crack propagation at all other times is assumed to take place in a principal plane as close as possible to the dominant crack propagation direction, irrespective whether the corresponding principal self-similar stress is the maximum one at that time or not. This means that for all times t we are looking for $j \in \{1,2,3\}$ such that

$$\left|\vec{n}^{\,0}_j(t)\cdot\vec{n}(t_{\max})\right| \geq \left|\vec{n}^{\,0}_i(t)\cdot\vec{n}(t_{\max})\right|, i \in \{1,2,3\}, i \neq j. \tag{3}$$

The selected direction and the corresponding equivalent K-factor are called $\vec{n}^{\,s}(t) \equiv \vec{n}^{\,0}_j(t)$ and $K^s_{eq}(t) \equiv \sigma^0_j(t)$, respectively. Summarizing, after determining a dominant crack propagation direction based on the maximum crack propagation rate (looking at each time point separately, as if evaluating a 0-max cycle), the propagation direction (and equivalent K-factor) for all other time points is determined based on its proximity to the dominant direction.

Cycle extraction

Now, the crack propagation rate based on the selected equivalent K-factor is plotted as a function of time (Fig. 3). Notice that the material constants are temperature-dependent and that the correct time-dependence of the temperature has to be taken into account. A subsequent cycle extraction, e.g. performed with the algorithm explained in [3] yields a upper and a lower data point for each extracted cycle. Denoting the corresponding times by t_1 and t_2, these data points are characterized by a selected equivalent K-factor and a temperature: $\{K^s_{eq}(t_1), T(t_1)\}$ and $\{K^s_{eq}(t_2), T(t_2)\}$. To evaluate the crack propagation due to this cycle, a K-factor range is defined by $\Delta K_{eq} = K_{eq}^{\max} - K_{eq}^{\min}$ based on the maximum value $K_{eq}^{\max}$ and minimum value $K_{eq}^{\min}$ of the selected K-factors $K_{eq}^{s}(t_1)$ and $K_{eq}^{s}(t_2)$. The temperature $T_{\max}$ is the maximum of the temperatures at the upper and lower cycle data point. Substituting ΔK_{eq} and $T_{\max}$ into Eq. 2 yields the crack propagation rate for the extracted cycle. The influence of the R-ratio ($K_{eq}^{\min} / K_{eq}^{\max}$), the threshold value and the critical value can be taken into account by extra multiplicative terms in Eq. 2. The crack propagation rate for the mission is the sum of the crack propagation rate of all extracted cycles. The corresponding bending and twist angle is $\varphi(t_{\max})$ and $\psi(t_{\max})$, respectively.

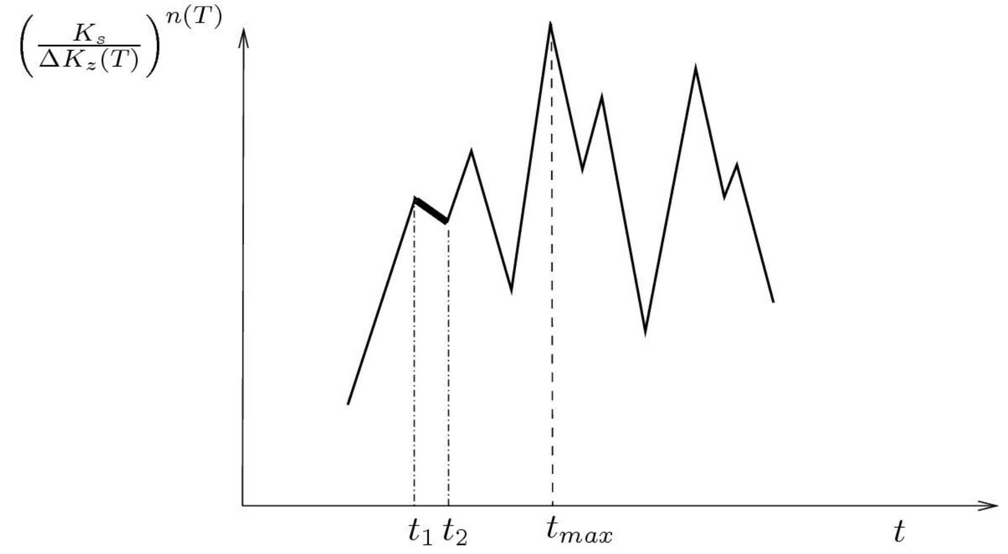

Figure3: Cycle extraction

Summary

Calculating crack propagation due to mixed-mode for complete missions introduces extra difficulties due to the change of the mixed-mode state during the mission. The present paper has presented a straightforward algorithm how to circumvent these problems. It is based on the identification of a dominant crack propagation direction and the calculation of the equivalent K-factors at all other times based on proximity of the principal self-similar stress planes. A subsequent cycle extraction and summation of the individual crack propagation rates leads to the crack propagation rate, bending angle and twist angle for the complete mission.

References

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